MAJOR TOPIC : RIEMANNIAN GEOMETRY

PART - I (General Theory)

Definition of Riemannian metrics.
Riemannian Length and Distance.
Space forms/Model spaces.
The Levi-Civita Connection. Koszul Formula.

Three fundamental curvature equations in Riemannian geometry : radial, tangential, and mixed.

Examples of Riemannian manifolds and computing curvatures : Spheres, products of spheres, warped products,
Lie groups, Riemannian submersions. CP^n.

(Riemannian) Geodesics. Examples : geodesics of the space forms and Lie groups with a bi-invariant metric.
Local and global uniqueness of geodesics. Exponential Map. Normal and uniformly normal neighborhoods.

The First Variation of Arc Length and Energy. Characterization of geodesics as the critical point of the the Length and Energy functional.
Gauss lemma and its consequences. Minimizing geodesics (segments). "Short" geodesics are segments. Completeness, Hopf-Rinow theorem. Cut locus,segment
domain, characterization of cut-locus, injectivity radius. Examples : cut-locus of model spaces, real projective spaces.

Isometries. Examples : isometries of space forms.
Jacobi Fields and Curvature. Jacobi fields on space forms. Conjugate points.
Second Variation of Arc Length, Energy and Convexity. Bonnet Myers theorem.
Parallel Transports.
Global Effects of Curvature. Bonnet-Myers, Synge-Weinstein and Cartan-Hadamard theorems. Applications.

PART - II (Comparison Theory)

Sectional curvature comparison I :

Manifolds of non-positive sectional curvature. Fundamental groups in non-positive curvature.
Cartan's theorem, Priessman's theorem (statements and corollaries, no techniques of proofs)

Comparison estimates : Riccati estimates, Hessian estimates.

Ricci curvature comparison :

Estimates for volume form , Hessian and Laplacian in comparison with space forms. Volume and relative volume
comparison, maximal diameter rigidity; Cheng's result.

Ricci curvature and fundamental group and 1st homology groups for compact manifolds : estimates of Betti numbers
(Gallot and Gromovs' theorems), structure of fundamental groups in compact manifolds.

Rays and lines .Cheeger gromoll splitting theorem, its corollaries : structure of universal cover of a complete
compact manifold with non-negative Ricci curvature.

MINOR TOPIC : RIEMANN SURFACES

Definitions : Riemann Surfaces. Examples of maps between some particular Riemann Surfaces.

Smooth surfaces, cotangent spaces and 1-forms. 2-forms and integrations. De Rham cohomology for surfaces.

Calculus on Riemann surfaces surfaces : Decomposition of the 1 forms (as a particular case of Hodge decomposition). Laplace operator and Harmonic functions. The Dirichlet norm.

Some basic analysis : Fourier transform of L^1 and L^2 functions, its properties, convolution, their relations.

Main theorem for compact Riemann surfaces : a 2 form with a zero integral is in the image of the Laplacian operator on smooth functions. Consequences of the theorem.

Proof of the main theorem using Riesz representation theorem, Weyl's lemma,

Uniformization theorem : Statement, analogue of the main thorem stated before for non-compact Riemann surfaces. The uniformization theorem, classificatiopn of Riemann surfaces.

Definition of moduli spaces and some simple examples using classification theorem.