Syllabus for Oral Examination

Thomas Sznigir

Partial Differential Equations:

Poisson's Equation: Fundamental solution, mean value property, maximum principle, Green's functions, Poisson's integral formula, Harnack's inequality, Liouville's theorem, Perron's method, single and double layer potentials, regularity

Heat Equation: Fundamental solution, maximum principle, uniqueness, energy estimate, Duhamel's principle, mean-value formula, regularity

Wave Equation: D'Alembert's solution, method of spherical means, energy methods, Duhamel's principle, domain of dependence

Sobolev Spaces: weak derivatives, approximation by smooth functions, extensions, compact embedding, Morrey's inequality, Gagliardo-Nirenberg inequality, Poincare inequality

Second-Order Elliptic Equations: weak solutions, Lax-Milgram theorem, interior and boundary regularity of solutions, maximum principle, Harnack inequality

Functional Analysis

Banach Spaces: Hahn-Banach theorem, Baire Category theorem and its consequences, linear functionals, dual space, weak and weak* topologies, reflexivity, separability

Hilbert Spaces: inner products, projection, Riesz Representation theorem, Lax-Milgram theorem, orthogonality, orthonormal bases

Compact Operators: definition, adjoints, spectral properties, Fredholm alternative, spectral decomposition of self-adjoint operators

Fourier Transform: definition and properties, inversion, convolution, Riemann-Lebesgue lemma, Plancherel's theorem, Parseval's formula, Hausdorff-Young inequality, tempered distributions.

References:

H. Brezis, Functional Analysis, Sobolev Spaces, and Partial Differential Equations
L. Evans, Partial Differential Equations
D. Gilbarg and N. Trudinger, Elliptic Partial Differential Equations of Second Order
E. Stein and G. Weiss. Introduction to Fourier Analysis on Euclidean Spaces