Syllabus for J. Tedor's Oral Qualifying Exam

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I. **Complex Analysis**

1. The $\overline{\partial}$ equation in one variable
   a. Cauchy's integral formula in discs and its applications
   b. The Runge approximation theorem
   c. The Mittag-Leffler theorem
   d. The Weierstraß theorem
2. The $\overline{\partial}$ equation in several variables
   a. Cauchy's integral formula in polydiscs and its applications
   b. Hartogs' extension theorem
3. Domains of holomorphy
   a. Holomorphic convexity
   b. Reinhardt domains and convergence of power series
4. Pseudoconvexity and plurisubharmonic functions
   a. The plurisubharmonicity of $-\log \delta(z, {}^c\Omega)$, Ω a domain of holomorphy
   b. Plurisubharmonic exhaustion functions
   c. Plurisubharmonic convexity
   d. The Levi form
   e. Analytic discs and the continuity principle
5. L^2 estimates and existence theorems for the $\overline{\partial}$ operator
   a. Hörmander's solution and the basic estimate
   b. The Levi problem

(Reference: Chapters 1, 2, 4 of Lars Hörmander, *An Introduction to Complex Analysis in Several Variables*)

II. **Harmonic Analysis**

1. The Fourier transform
   a. The L^1 theory and the inversion theorem
   b. The L^2 theory and the Plancherel theorem
   c. The class of tempered distributions
   d. Operators that commute with translations
2. Real-variable theory
   a. The Hardy-Littlewood maximal function
   b. The Lebesgue differentiation theorem
   c. The integral of Marcinkiewicz
   d. The Calderón-Zygmund decomposition
3. Basic interpolation theorems
   a. The Marcinkiewicz interpolation theorem
   b. The Riesz-Thorin interpolation theorem
4. Singular integrals of convolution type
   a. The gradient condition
   b. The Hörmander condition
   c. Homogeneous singular integrals
5. Examples of the theory
   a. The Hilbert transform
   b. The Riesz transforms
   c. Poisson integrals

(References: Chapter one of Elias Stein and Guido Weiss, *Introduction to Fourier Analysis on Euclidean Spaces* and chapters 1–3 of Elias Stein, *Singular Integrals and Differentiability Properties of Functions*)