

Oral Qualifying Examination Syllabus
Thuy Pham (December, 2003)

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1. COMMUTATIVE ALGEBRA

1.1. **Basic Constructions.**

- Localization
- Associated primes and primary decomposition
- Integral dependence and the Nullstellensatz
- Filtrations and the Artin-Rees lemma
- Completions and Hensel's lemma
- Valuation rings (DVRs - Dedekind rings - Krull rings)

1.2. **Dimension Theory.**

- Dimension zero
- Principal Ideal Theorem and system of parameters
- Noether Normalization and finiteness of integral closure

1.3. **Hilbert Functions of Graded Modules.**

- Graded rings and modules
- Hilbert functions over homogeneous rings
- Macaulay's theorem on Hilbert functions
- Gotzmann's regularity and persistence theorem
- Filtered rings
- The Hilbert-Samuel function and reduction ideals
- Multiplicity

1.4. **Cohen-Macaulay Rings.**

- Regular sequences
- Grade and depth of a module
- Depth and projective dimension
- Cohen-Macaulay rings and modules
- Regular rings and normal rings
- Complete intersection

1.5. **Canonical Modules.**

- Finite modules of finite injective dimension
- Injective hulls - Matlis duality
- The canonical module of a local ring
- The canonical module of a graded ring

2. HOMOLOGICAL ALGEBRA

2.1. **Category Theory Language.**

- Categories
- Functors
- Adjoint functors
- Abelian categories

2.2. **Chain Complexes.**

- Chain homotopies
- Mapping cones and cylinders

2.3. **Derived Functors.**

- Projective and Injective resolutions
- Left and Right derived functors
- Tor and Ext

2.4. **Homological Dimensions.**

- Rings of small dimensions
- Change of ring theorems
- Koszul complexes
- Local cohomology

2.5. **Spectral Sequences.**

- Spectral sequence of a filtration
- Convergence
- Spectral sequence of a double complex

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