

Syllabus for Oral Examination  
Tian Yang (May, 1st, 2009)

Algebraic and Differential Topology

The Fundamental Group  
The Seifert-Van Kampen Theorem  
Covering Spaces  
Lifting properties  
Classification of covering spaces  
Deck Transformations and group actions

Simplicial homology  
Singular homology  
Homotopy Invariance  
Exact Sequence and Excision  
Cellular Homology  
Mayer-Vietoris Sequence

Cohomology ring  
Künneth formula  
Cup and Cap Products  
Poincare Duality

Smooth Manifolds  
Tangent and Cotangent Spaces  
Differential Forms  
Operators on Differential Forms:  
Contractions, Lie-derivatives and Exterior Differentials  
de Rham Cohomology

Reference  
Allen Hatcher, Algebraic Topology  
James Vick, Homology Theory: An Introduction to Algebraic Topology  
John Lee, Smooth Manifolds

## Riemannian Geometry

Riemannian metrics

Levi-Civita connection

Parallel translation

Curvature tensor

Sectional curvature, Ricci curvature, Scalar curvature

Geodesic, Exponential map, Gauss lemma

Riemannian manifolds as metric spaces

Hopf-Rinow theorem

First and second variations of arc length

Jacobi fields

Manifolds with constant sectional curvature

Cartan-Hadamard theorem

Bonnet-Myers theorem

Reference

Karsten Grove, Riemannian Geometry: A Metric Entrance Peter

Petersen, Riemannian Geometry