Oral Qualifying Exam Syllabus

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1. Combinatorics and Graph Theory

1.1. Combinatorics

Basics: counting arguments, generating functions, binomial coefficients, recurrence relations, inclusion-exclusion, Stirling's formula.

Set Systems: Sperner's Theorem, LYM inequality, Erdos-Ko-Rado, Dilworth's Theorem, statement of Kruskal-Katona

Lattices: geometric and distributive lattices, chains in distributive lattices, linear extensions of posets, the Mobius inversion formula, Weisner's Theorem, Birkhoff representation theorem

Ramsey Theory: Ramsey's Theorem for graphs and hypergraphs, infinite Ramsey, Konig's Lemma, upper and lower bounds, probabilistic lower bounds, statements of Hales-Jewett and van der Waerden

1.2. Graph Theory

Matching: Hall's theorem, Konig's theorem, matching algorithms, Augmenting Paths, Tutte's Theorem.

Connectivity and Spanning Trees: Menger's Theorem, Max Flow/Min Cut Theorem, Prim's Algorithm, Kruskal's Algorithm, Dijkstra's Algorithm, Matrix Tree Theorem, Cayley's formula, Prufer Codes.

Planarity: Euler's Formula, Proof that K5 and K3,3 are not planar, Kuratowski, Wagner's Theorem, Crossing number.

Coloring: Chromatic and Edge Chromatic Numbers, List Coloring, Brook's Theorem, Konig's Line Coloring Theorem, Vizing's Theorem, Thomassen's Theorem, 5-color theorem, Galvin's Theorem, perfect graphs: definition and statements of theorems

Extremal Problems: Turan's Theorem, Statement of Regularity Lemma and its application to the Erdos-Stone Theorem.

1.3. Probabilistic Methods

Basics: Linearity of Expecation, Markov's Inequality, Chernoff bounds, Chebyshev Inequality, statement of Azuma's Inequality, binomial and Poisson distributions

Second Moment Method: thresholds of balanced graphs, clique number, distinct sums

Method of alterations: high girth and high chromatic number, R(k,k), independence number, lower bound on property B

Lovasz Local Lemma: Symmetric and general versions, application to Ramsey lower bounds.

Poisson Paradigm: Janson inequalities, Brun's sieve, threshold for EPIT

2. Computational Complexity

P v. NP: Definitions, reducibility, the Cook-Levin Theorem, NP completeness of SAT, independent set, 0/1 integer programming, and directed hamiltonian path, conditions that imply $P \neq NP$

Diagonalization: Ladner's Theorem, Oracle Turing Machines and the Baker-Gill-Solovay Theorem

Space-bounded complexity: definitions, PSPACE completeness of TQBF, NL completeness of PATH, Savitch's theorem, the Immerman-Szeleplcsenyi Theorem

Separation theorems: Time and Space Hierarchy Theorems (deterministic and nondeterministic versions)

Polynomial hierarchy: Definitions of Σ_i, Π_i, complete problems, conditions that lead to the collapse of PH.

Circuits: $P \subseteq$ P/poly, CKT-SAT and alternate proof of Cook-Levin, Characterization of P/poly as TMs with advice, Karp-Lipton Theorem, Meyer's Theorem, existence of hard functions, Nonuniform Hierarchy Theorem, definitions of NC_i, AC_i

Randomization: Definitions of RP, BPP and ZPP, error reduction, Sipser-Gacs Theorem, $BPP \subseteq$ P/poly, randomized reductions and definition of $BP \bullet NP$

Inteactive Proofs: definitions, dIP=NP, $GNI \in AM$, NP completeness of GI implies $\Sigma_2 = \Pi_2$, IP=PSPACE

PCP theorem: definitions, equivalence of the 3 versions, hardness of approximation for vertex cover and independent set, $NP \subseteq PCP(poly(n), 1)$

Decision Trees: Decision tree complexity, 0- and 1-certificates, certificate complexity, randomized decision tree complexity, sensitivity, block sensitivity, degree, relationships between s(f),bs(f),C(f),D(f),deg(f),R(f)

Communication Complexity: Fooling sets, tiling lower bound, rank lower bound, discrepancy, eigenvalue bound, $\varepsilon(f)$, multiparty communication complexity, $GIP_{k,n}$

Lower bounds: Hastad's switching lemma, parity $\notin AC0$, Razborov-Smolensky theorem, sunflower lemma, monotone-circuit lower bound for CLIQUE