Topics for oral qualifying exam for Tom Robinson
Spring, 2006

Major topic: Vertex operator algebras

1. Definitions and properties.
   (a) Formal calculus.
   (b) The notions of vertex algebra and of vertex operator algebra, and basic properties.
   (c) Rationality, commutativity and associativity; equivalence of various formulations, including "weak" formulations.

2. Representations of vertex (operator) algebras.
   (a) The notion of module and basic properties.
   (b) Weak vertex operators.
   (c) The structure of the canonical weak vertex algebra. Local subalgebras and vertex subalgebras of the canonical weak vertex algebra.
   (d) The equivalence between modules and representations.
   (e) General construction theorems for vertex (operator) algebras and modules.

3. Examples of vertex (operator) algebras and modules.
   (a) Vertex (operator) algebras and modules based on the Virasoro algebra.
   (b) Vertex (operator) algebras and modules based on affine Lie algebras.
   (c) Vertex (operator) algebras and modules based on Heisenberg Lie algebras.
   (d) Vertex (operator) algebras and modules on even lattices.
   (e) Vertex operator construction of the affine Lie algebras corresponding to A_n, D_n and E_n.

4. The fundamentals of logarithmic tensor product theory for generalized modules for a conformal vertex algebra.
    (a) The notions of conformal and Möbius vertex algebra.
    (b) The notions of module and of generalized module.
    (c) Opposite vertex operators, and contragredient modules and generalized modules.
    (d) The notions of intertwining operator, logarithmic intertwining operator and fusion rule; basic properties.
    (e) Logarithmic formal calculus.
    (f) The notion of $P(z)$-tensor product of generalized modules.

Minor topic: Lie algebras

1. Elementary notions and basic theory
    (a) Definitions, examples, representations, modules
    (b) Solvable, nilpotent, simple and semisimple Lie algebras and the Killing form
    (c) Lie's theorem
    (d) Engel's theorem
    (e) Cartan subalgebras
    (f) Cartan's criteria for semisimplicity and solvability
    (g) Semisimple Lie algebras as direct products of simple Lie algebras
    (h) Complete reducibility of modules for semisimple Lie algebras
2. Semisimple Lie algebras and root systems
    (a) Representations of $sl(2)$
    (b) Root space decompostion
    (c) Axiomatics of root systems; simple roots; Weyl group
    (d) Classification
    (e) Construction of root systems

3. Universal enveloping algebras
    (a) Construction of the universal enveloping algebra
    (b) The Poincaré-Birkhoff-Witt theorem
    (c) Free Lie algebras

References

[FHL] I. Frenkel, Y.-Z. Huang and J. Lepowsky, On Axiomatic Approaches to Vertex Operator Algebras and Modules, Memoirs Amer. Math. Soc. 104 (1993).

[FLM] I. Frenkel, J. Lepowsky and A. Meurman, Vertex Operator Algebras and the Monster, Academic Press, 1988.

[HLZ] Y.-Z. Huang, J. Lepowsky and L. Zhang, Logarithmic tensor product theory for generalized modules for a conformal vertex algebra, to appear.

[H] J. Humphreys, Introduction to Lie Algebras and Representation Theory, Second Printing, Revised, Springer-Verlag, 1972.

[LL] J. Lepowsky and H. Li, Introduction to Vertex Operator Algebras and Their Representations, Progress in Math., Vol. 227, Birkhäuser, Boston, 2003.

[LM] J. Lepowsky and G. McCollum, Elementary Lie algebra theory, Yale Univ. Lecture Note Series, 1974, 138 pgs.; corrected, 1982, Rutgers Univ.