Oral Qualifying Exam Syllabus

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Elliptic Curves and Algebraic Geometry

1. Elliptic Curves
   (a) Elliptic Curves over $\mathbb{C}$
   (b) Elliptic Curves over finite fields, The Hasse Bound
   (c) Elliptic Curves over local fields
   (d) Elliptic Curves over global fields, The Mordell-Weil Theorem
2. Algebraic Geometry
   (a) Sheaves, Schemes, First Properties, Sheaves of Modules
   (b) Divisors, Riemann-Roch for Curves
   (c) Differentials, Genus
   (d) Derived Functors, Cohomology of Sheaves, Cohomology of Noetherian Affine Schemes and Projective Space, Serre Duality

Algebraic Number Theory and Modular Forms

1. Algebraic Number Theory
   (a) Number Fields: Integral Bases, Different, Discriminant
   (b) Ramification, Splitting of primes
   (c) Ideal Class Group, Minkowski's Bound, Finiteness of class number
   (d) Dirichlet's Unit Theorem
2. Modular Forms
   (a) Modular Forms for the full modular group and its congruence subgroups
   (b) Eisenstein Series
   (c) Structure of the ring of modular forms
   (d) Hecke Operators