Syllabus for the Oral Qualification Exam

Ved V. Datar

Committee : Jian Song (chair), Sagun Chanillo, Xiaojun Huang, Xiaowei Wang

I. Complex Geometry

1. Cěch Cohomology:

- Sheaves and Cohomology Groups
- Exact sequences
- Leray's theorem (Only the statement)
- Poincare's and Dolbeault's Lemmas
- The de Rham and Dolbeault isomorphisms

2. Compact Hermitian Manifolds:

- The Hodge theorem
- Kodaira-Serre duality
- The Kähler condition and equivalent characterizations
- Hodge identities and Hodge decomposition for Kähler Manifolds
- The $\partial\overline{\partial}$ Lemma on Kähler Manifolds
- Curvature Computations on a Kähler manifold

3. Kodaira's Embedding Theorem:

- Line bundles and Divisors
- Adjunction formulas
- Vanishing theorems
- Hodge manifolds and embedding

II. Complex Analysis in Several Variables

1. Basic Theory

- Cauchy integral formula in polydiscs, Cauchy estimates
- Hartog's extension Theorem
- Domains of holomorphy, holomorphic Convexity and the theorem of Cartan and Thullen.
- Continuity Principle for domains of holomorphy
- Zeroes of a holomorphic function ; Weierstrass theorems

2. Pseudoconvexity:

- Properties of subharmonic functions, plurisubharmonic functions;
- Pseudoconvexity and Levi pseudoconvexity;
- The Narasimhan Lemma for Strictly Pseudoconvex domains
- Oka's Theorem.
- Equivalent characterizations of Pseudoconvexity

3. The Levi problem:

- L^2 theory for the $\bar{\partial}$ operator
- Solution to the Levi problem.

III. Additional Topic

- Harnack inequality for elliptic equations in divergence form

References

[M-K] Morrow, Kodaira, Complex Manifolds.

[G-H] Griffiths, Harris , Principles of Algebraic Geometry

[HOR] Lars Hörmander, An introduction to Complex Analysis in several variables.

[RAN] Michael Range, Holomorphic funtions and Integral Representations in Several Complex Variables

[MOS] Moser, Jürgen (1961), "On Harnack's theorem for elliptic differential equations", Communications on Pure and Applied Mathematics 14: 577–591