

Syllabus for Vijay

Algebraic Geometry (major topic)

1) Affine, projective, quasi-projective varieties

Zariski topology, ideals of varieties, coordinate ring

Ring of regular functions ($= A(X)[1/f]$ of U_f)

Local ring at a point

Regular maps

Examples: Veronese varieties, Segre varieties, sub-varieties of these

Product of varieties (is actually categorical product)

2) Cones

Classification of quadrics

Projections (proof that these are varieties)

Morphisms are closed maps

3) Nullstellensatz (Weak and Strong)

4) Grassmannians

Plucker embedding

Subvarieties of Grassmannians (incident planes, joins, fano varieties,

Schubert Varieties)

5) Rational functions and rational maps

Graphs of rational maps

Every variety is birational to a hypersurface

Blowups, blowdowns

6) Dimension

Equivalence of various definitions

Basic computations (Grassmannians, products, cones, projections)

Intersection of a variety with a hypersurface

Fiber Dimension Theorem

Complete intersections

Dimensions of secant varieties, joins, flag manifolds, Schubert varieties

7) Hilbert Functions

$h(m)$ is a polynomial for large m

Degree of polynomial = dimension of variety

8) Smoothness, Tangent Spaces

Variety is singular at point iff local ring is regular local ring

Singular locus of a variety is proper closed subset

Singular points of Schubert varieties

9) Sheaves, Sheafification, Etale Space

Schemes: Spec, Proj, and the Functor from Varieties to Schemes

References

Joe Harris, Algebraic Geometry: A First Course

Robin Hartshorne, Algebraic Geometry

Algebraic Topology (minor topic)

1) The Fundamental Group
The Seifert-Van Kampen Theorem
Classification of covering spaces
Deck Transformations and group actions

2) Simplicial homology
Singular homology
Exact Sequence and Excision
Cellular Homology
Mayer-Vietoris Sequence

3) Cohomology ring
Kunneth formula
Cup Product
Statement of Poincare Duality

4) Vector bundles
Grassmann manifolds
Universal Bundles, Gauss Maps

References
Allen Hatcher, Algebraic Topology
Milnor, Stasheff, Characteristic Classes