

# Oral Exam Syllabus

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## 1 Combinatorics

**Basic Enumeration:** counting arguments, generating functions, recurrence relations, reflection principle, inclusion-exclusion

**Permutations & Tableau:** Robinson-Schensted Correspondence, Knuth relations, Greene's Theorem, the Hook Length Formula, the Marcus-Tardos Theorem (Stanley-Wilf Conjecture), generating trees, WQO for permutations

**Hypergeometric functions & identities:** definition and representation, notion of  $q$ -analog and examples, Sister Celine's Algorithm, Gosper's Algorithm, Zeilberger's Algorithm

**Extremal Set Theory:** Ray-Chaudhuri Wilson and variants, Sperner's theorem, Erdős-Ko-Rado, Borsuk conjecture and Kahn-Kalai counterexample, Harper's Theorem, Kruskal-Katona

**Lattices & Posets:** Dilworth's Theorem, Fundamental Theorem of Finite Distributive Lattices, Birkhoff covering property, Jordan-Dedekind chain condition, Möbius inversion, Weisner's Theorem, Dowling-Wilson Theorem

**Correlation Inequalities:** Harris-Kleitman, FKG, four functions, application to  $xyz$  inequality, BK inequality, stochastic domination

**Ramsey Theory:** Ramsey's Theorem, infinite Ramsey theory, probabilistic lower bounds, stepping-up lemma, van der Waerden,  $R(3, n)$  upper and lower bounds

**Discrepancy:** Erdős-Selfridge Theorem, Beck-Fiala Theorem, discrepancy in arithmetic progressions, linear and hereditary discrepancy, 6 standard deviations suffice, Komlós Conjecture

**Algebraic Methods:** Combinatorial Nullstellensatz, Chevalley-Waring Theorem, Cauchy-Davenport Theorem, Frankl-Wilson Theorem, bounds on the Shannon capacity using dimension arguments

## 2 Graph Theory

**Matching:** Hall's Theorem, bipartite matching algorithm, König's Theorem, Tutte's 1-factor Theorem, matching polytope

**Connectivity:** Menger's Theorem, Max-Flow-Min-Cut Theorem, Kruskal's Algorithm

**Planarity:** Euler's formula, Kuratowski's Theorem, Wagner's Theorem

**Coloring:** Brooks' Theorem, Vizing's Theorem, 5 Color Theorem, perfect graphs, Weak Perfect Graph Theorem

**Extremal Problems:** Turán's Theorem, statement of Regularity lemma and application to Erdős-Stone Theorem

## 3 Probabilistic Methods

**Basics:** Stirling's formula, Bonferroni inequalities, Chebyshev's inequality, Chernoff bound

**Alterations:** application to lower bound on property B

**Second Moment Method:** general procedure, application to threshold function for having a certain graph as a subgraph,  $2^{nd}$  moment method for nonnegative random variables

**Lovász Local Lemma:** symmetric and general versions, applications to linear arboricity conjecture and Latin transversals

**Martingales:** Azuma's inequality, edge and vertex exposure, applications to chromatic number

**Poisson Paradigm:** Janson inequalities, application to number of triangles in  $G_{n,p}$ , Brun's sieve, application to number of isolated points

**Random graphs:** monotone properties,  $G_{n,p}$  versus  $G_{n,M}$ , existence of threshold functions, relationship between connectedness and having no isolated vertices, probabilistic refutation of Hajós's Conjecture

## 4 Probability

Probability Spaces

Random Variables

Borel-Cantelli Lemma

Laws of large numbers

Law of iterated logarithm

Central Limit Theorem

Conditional Expectation

Random Walks & Polya's Theorem

Arc-Sine Laws

Kolmogorov's Zero-One Law

Are you paying attention?

**Percolation Theory:**

Bond percolation

Continuity properties of  $\theta(p)$

Uniqueness of infinite cluster in  $L^d$ .