Topics for oral qualifying exam for Zhixiang Wang
Spring, 2003

Major topic: Vertex operator algebras

The general theory of vertex operator algebras, basic examples, the basic geometry of vertex operator algebras as presented in [FLM] (Chapters 1–8), [FHL], [H] and [LL]. The specific topics are:

1. Definitions and properties.
    (a) Formal calculus.
    (b) The notions of vertex algebra and of vertex operator algebra, and basic properties.
    (c) Rationality, commutativity and associativity (various formulations, including "weak" formulations.
2. Examples of vertex (operator) algebras.
    (a) Vertex (operator) algebras based on Heisenberg Lie algebras.
    (b) Vertex (operator) algebras based on affine Lie algebras.
    (c) Vertex (operator) algebras on the Virasoro algebra.
    (d) Vertex (operator) algebras on even lattices.
3. Representations of vertex (operator) algebras.
    (a) The notion of module and basic properties.
    (b) The relations between this notion and the topics listed above, including the construction of modules from mutually local operators.
    (c) The notions of intertwining operator and fusion rule, basic properties.
    (d) Examples of modules for the vertex operator algebras listed above.
4. The geometry of vertex operator algebras.
    (a) The moduli spaces of spheres with tubes, the sewing operation, examples for the sewing operation.

(b) The geometric interpretations of vertex operators and vacua, the geometric meanings of commutativity, associativity, skew-symmetry, $L(-1)$-derivative property, vacuum property and creation property.

(c) The notion of geometric vertex operator algebra and the statement of the isomorphism theorem.

(d) Determinant lines of Fredholm operators, determinant lines over Riemann surfaces with parametrized boundaries, relation to the central charges of vertex operator algebras.

Minor topic: Riemann geometry

Basic Riemannian geometry as presented in the first six chapters of [P]. The specific topics are:

1. Riemannian metrics
   - (a) Riemannian manifolds and maps.
   - (b) Groups and Riemannian manifolds.
   - (c) Local representations of metrics.
   - (d) Doubly warped products.
2. Curvature
   - (a) Connections and curvature.
   - (b) The fundamental curvature equations and the equations of Riemannian geometry.
3. Examples
   - (a) Warped products.
   - (b) Hyperbolic space.
   - (c) Left-invariant metrics.
   - (d) Complex projective space.
4. Hypersurfaces
   - (a) The Gauss map
   - (b) Existence of hypersurfaces.
   - (c) The Gauss-Bonnet theorem.

5. Geodesics and distance
    (a) The connection along curves and geodesics.
    (b) The metric structure of a Riemannian manifold.
    (c) The exponential map.
    (d) Short geodesics are segments.
    (e) Local geometry in constant curvature.
    (f) Completeness.
    (g) Characterization of segments
    (h) Metric characterization of maps.
6. Sectional curvature comparison
    (a) Basic comparison estimates.
    (b) Riemannian covering maps.
    (c) Positive sectional curvature.

References

[FHL] I. Frenkel, Y.-Z. Huang and J. Lepowsky, On Axiomatic Approaches to Vertex Operator Algebras and Modules, Memoirs Amer. Math. Soc. 104 (1993).

[FLM] I. Frenkel, J. Lepowsky and A. Meurman, Vertex Operator Algebras and the Monster, Academic Press, 1988.

[H] Y.-Z. Huang, Two-dimensional conformal geometry and vertex operator algebras, Progress in Math., Vol. 148, Birkhäuser Boston, 1997.

[LL] J. Lepowsky and H. Li, Introduction to vertex operator algebras and their representations, to appear.

[P] P. Petersen, Riemannian geometry, Graduate Text in Mathematics, Springer-Verlag, New York, 1997.