

Oral Exam

Bill Cuckler

October 7, 2003

Committee: Profs J. Beck, J. Kahn, J. Komlós, M. Saks

1 Combinatorics and Graph Theory

1.1 Combinatorics

Basic Enumeration: counting arguments, generating functions, recurrence relations, reflection principle, inclusion-exclusion principle

Extremal Set Theory: Ray-Chaudhuri Wilson, Frankl Wilson, Sperner's theorem, Erdős-Ko-Rado, Borsuk conjecture and Kahn Kalai counterexample, Harper, Kruskal Katona

Lattices: Geometric and distributive lattices, Birkhoff covering property, Jordan-Dedekind chain condition, Möbius inversion, Weisner's theorem, Dowling-Wilson theorem

Correlation Inequalities: Harris-Kleitman, FKG, four functions, application to xyz inequality, BK inequality, stochastic domination

Discrepancy: Erdős-Selfridge theorem, Beck-Fiala, discrepancy in arithmetic progression, linear and hereditary discrepancy, 6 standard deviations suffice, Komlós conjecture

Ramsey Theory: Ramsey's theorem, infinite Ramsey, probabilistic lower bounds, stepping-up lemma, van der Waerden, $R(3, n)$ upper and lower bounds

1.2 Graph Theory

Matching: Hall's theorem, bipartite matching algorithm, König's theorem, Tutte's 1-factor theorem, matching polytope

Connectivity: Menger, Max-Flow-Min-Cut theorem, Dilworth's theorem, Kruskal's algorithm

Planarity: Euler's formula, Kuratowski's theorem, Wagner's theorem

Coloring: chromatic and edge chromatic numbers, Brook's theorem, Vizing's theorem, 5 color theorem, perfect graphs, weak perfect graph theorem

Extremal Problems: Turán's theorem, statement of Regularity lemma and application to Erdős-Stone theorem

1.3 Probabilistic Methods

Basics: Stirling's formula, linearity of expectation, Bonferroni inequalities, coupling, Chebyshev's inequality, Chernoff bound

Alterations: application to lower bound on property B

Second Moment Method: general procedure, application to threshold function for having a certain graph as a subgraph, 2^{nd} moment method for nonnegative random variables

Lovász Local Lemma: symmetric and general versions, applications to linear arboricity conjecture and Latin transversals

Martingales: Azuma's inequality, edge and vertex exposure, applications to chromatic number

Poisson Paradigm: Janson inequalities, application to number of triangles in $G_{n,p}$, Brun's sieve, application to number of isolated points

Random graphs: monotone properties, $G_{n,p}$ versus $G_{n,M}$, existence of threshold functions, relationship between connectedness and having no isolated vertices, probabilistic refutation of Hajós conjecture

2 Probability Theory

Probability Spaces

Random Variables

Borel-Cantelli Lemma

Laws of large numbers

Law of iterated logarithm

Central Limit Theorem

Conditional Expectation

Random Walks and Polya's theorem

Arc-Sine Laws

Maximal ergodic theorem and Birkhoff Ergodic Theorem

Application of ergodic theorem to equidistribution

Kolmogorov's Zero One Law

Percolation Theory:

Bond percolation

Continuity properties of $\theta(p)$

Uniqueness of infinite cluster in L^d