

Oral Qualifying Exam Syllabus

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1 Committee

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2 Algebraic Topology and Cobordism Theory

2.1 Algebraic Topology

1. The Fundamental Group (the van Kampen theorem, covering spaces)
2. Homology
 - (a) Simplicial, singular, and cellular homology
 - (b) Simplicial approximation
 - (c) Homology theories
3. Cohomology
 - (a) Simplicial, singular, cellular, de Rham cohomology
 - (b) Cohomology with local coefficients, presheaf cohomology
 - (c) Cup, cap, and cross products
 - (d) Thom isomorphism theorem
 - (e) Künneth Formulas
 - (f) Poincaré duality, Alexander duality, Lefschetz duality
 - (g) Universal Coefficients for Homology
 - (h) Cohomology theories
4. Homotopy Theory
 - (a) Whitehead's theorem
 - (b) Cellular approximation theorem, CW approximation theorem
 - (c) The Hurewicz theorem
 - (d) Fibrations, long exact sequence, Gysin sequence
 - (e) Homotopy groups of spheres, stable structure, Freudenthal suspension theorem
 - (f) Homotopy construction of cohomology
5. Generalized cohomology and homology theories
 - (a) Spectra, Eilenberg MacLane spectra
 - (b) The Brown Representability Theorem
 - (c) Steenrod algebra, cohomology operations

2.2 Cobordism Theory

1. h -cobordism theorem
2. (B, f) manifolds and cobordisms
3. Computation of MU_* , MO_* , MSO_*
4. Stiefel-Whitney classes and Pontrjagin classes
5. Oriented cohomology theories

2.3 References

1. Allen Hatcher, *Algebraic Topology*.
2. Raoul Bott; Loring Tu, *Differential Forms in Algebraic Topology*.
3. Davis & Kirk, *Lecture Notes in Algebraic Topology*.
4. Vick, *Homology Theory*.
5. J. P. May, *A Concise Course in Algebraic Topology*.
6. Tom Weston, *An Introduction to Cobordism Theory*.
7. Milnor & Stasheff *Characteristic Classes*.
8. Stong, *Notes on Cobordism Theory*.
9. J. F. Adams, *Stable Homotopy and Generalised Homology*

3 Homological Algebra and Category Theory

3.1 Homological Algebra

1. Chain complexes of modules
2. Derived functors
3. Tor, Ext, and \lim^1
4. Universal coefficient theorems
5. Spectral sequences
6. Hyperhomology, Grothendieck spectral sequence, exact couples

3.2 Category Theory

1. Adjoint functors, adjoint functor theorems
2. (Co-)limits
3. Abelian categories
4. Simplicial sets
5. Monoidal categories
6. Model categories

3.3 References

1. Hilton & Stammach, *A Course in Homological Algebra*.
2. Saunders Mac Lane, *Categories for the Working Mathematician*.
3. Charles Weibel, *An Introduction to Homological Algebra*.
4. J. P. May, *Simplicial Objects in Algebraic Topology*.