Oral Qualifying Exam Syllabus

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I. Combinatorics, Graph Theory, and the Probabilistic Method.

1 Combinatorics

Basic Enumeration:
counting arguments ([vLW, Ch. 10])
stirling's formula ([F, p. 52–54])
sieves methods, inclusion-exclusion principle ([vLW, Ch. 10])
recurrence relations and generating functions ([Stan, Ch. 4], [vLW, Ch. 14])

Extremal Set Theory:
intersecting families, Erdős-Ko-Rado theorem([vLW, Ch. 6, p. 56])
LYM inequality and Sperner's theorem ([vLW, Ch. 6])
Littlewood-Offord problems, combinatorial and analytic approaches ([TV, Ch. 7], [VV])

Correlation Inequalities:
Harris-Kleitman theorem([AS, p. 86])
Fortuin-Kasteleyn-Ginibre (FKG inequality) ([AS, p. 84], [TV, p. 34])
Ahlswede-Daykin four functions theorem ([AS, p. 82])
application to Shepp's xyz inequality ([S], [AS, p. 88])

Ramsey Theory:
Ramsey's theorem ([vLW, p. 28], [BM, Ch. 7], [KJ])
Ramsey numbers([BM, Ch. 7], [KJ])
upper bounds and probabilistic lower bounds ([AS, p. 16, p. 25, p. 67])

2 Graph Theory

Matching:
König's Minimax theorem ([D, Thm 2.1.1], [KJ])
Hall's theorem ([D, Thm 2.1.2], [KJ], [BM, Ch. 9])
Application to Birkhoff-von Neumann theorem([KJ], [BM, Ch. 5])
Tutte's 1-factor theorem ([D, Thm 2.2.1], [KJ])

Coloring:
König edge coloring theorem for bipartite graphs ([D, Prop 5.3.1], [KJ], [BM, Ch. 5])
Brooks's theorem-vertex coloring ([D, Thm 5.2.4], [KJ], [BM, Ch. 8])

Vizing's Theorem-edge coloring ([D, Thm 5.3.2], [KJ], [BM, Ch. 6])
5 color theorem ([D, Prop 5.1.2], [KJ], [BM, Ch. 9])

Planarity:
Euler's formula ([D, Thm 4.2.9], [KJ], [BM, Ch. 9])
Kuratowski's theorem ([D, §4.4], [BM, Ch. 9])
Wagner's theorem ([D, Thm 4.4.6])

3 Probabilistic Methods

Basics:
linearity of expectation ([AS, Ch. 2.1])
Bonferroni inequalities
Normal, Binomial, and Poisson distributions ([DR])
Chernoff bound ([AS, Apdx. A], [TV, §1.3])

Second Moment Method:
Chebyschev's inequality ([AS, p. 41])
application to threshold function for having a certain graph as a subgraph ([AS, Ch. 4], [SJ, Lec. 3])

Alteration Method:
general procedure
application to high girth and high chromatic numbers([KJ], [SJ, Lec. 2])

Lovász Local Lemma:
symmetric and general versions ([AS, p. 64–65], [SJ, Lec. 8], [VV])
application to lower bounds of Ramsey numbers ([AS, §5.6])

Martingales and Tight Concentration:
Azuma's inequality ([AS, §7.2], [VV])
edge and vertex exposure martingales ([AS, §7.1])
application to concentration of chromatic number ([AS, §7.3])
general and combinatorial Talagrand's inequality ([AS, §7.5,§7.6], [VV])
application to independence number of $G_{n,1/2}$ ([AS, p. 110])

Poisson Paradigm:
Janson inequalities ([AS, §8.1], [TV, §1.6])
application to number of triangles in $G_{n,p}$ ([AS, §10.1])
Brun's sieve ([AS, §8.3])
application to number of isolated vertices in $G_{n,p}$ ([AS])

Random Graphs:
$G_{n,p}$ versus $G_{n,M}$
existence of threshold functions ([AS, p. 156–157])
connectedness (Erdős-Renyi)([SJ, Lec. 2])

II. Probability Theory

Probability Spaces/Random Variables:
algebra and sigma-algebra
probability spaces
monotone class theorem
independence and product spaces
random variables
distribution functions
expectation
independence of random variables
convergence concepts for random variables

Law of Large Numbers:
weak law of large numbers
Borel-Cantelli lemma
strong law of large numbers
Kolmogorov's 0-1 law
Kolmogorov's maximal inequality
Kolmogorov's three series theorem

Central Limit Theorem:
De Moivre-Laplace theorem
weak convergence and convergence in distribution
fourier transform, characteristic functions
multiplication formula
inversion formula, plancherel theorem
uniqueness theorem
continuity theorem
proofs of central limit theorem

Ergodic Theorem:
measure preserving transformations
Birkhoff's ergodic theorem

References

[AS] Alon, Noga; Spencer, Joel H. *The probabilistic method.* Second edition. Wiley-Interscience Series in Discrete Mathematics and Optimization. Wiley-Interscience [John Wiley & Sons], New York, 2000. xviii+301 pp.

[SJ] Spencer, Joel. *Ten lectures on the probabilistic method.* Second edition. CBMS-NSF Regional Conference Series in Applied Mathematics, 64. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 1994. vi+88 pp.

[TV] Tao, Terence, Vu, Van. *Additive Combinatorics.* Cambridge Studies in Advanced Mathematics, 105. Cambridge University Press, Cambridge, 2006. xviii+512 pp.

[D] Diestel, Reinhard. *Graph Theory.* Third edition. Graduate Texts in Mathematics, 173. Springer-Verlag, Berlin, 2005. xvi+411 pp.

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[Stan] Stanley, Richard P. *Enumerative combinatorics.* Vol. 1. Cambridge Studies in Advanced Mathematics, 49. Cambridge University Press, Cambridge, 1997. xii+325 pp.

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[DR] Durrett, Richard. *Probability: theory and examples.* Second edition. Duxbury Press, Belmont, CA, 1996. xiii+503 pp.

[VV] Vu, Van. notes from *Random and Pseudo-Random Structures*, Rutgers University, Fall 2008 (notes taken by Ke Wang).

[KJ] Komlos, János. notes from *Graph Theory*, Rutgers University, Spring 2008 (notes taken by Ke Wang).

[GR] Gundy, Richard. notes from *Adcanced Probability*, Rutgers University, Fall 2008 (notes taken by Ke Wang).