

SYLLABUS FOR ORAL QUALIFYING EXAM

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Major Topic: Partial Differential Equations

- Laplacian Equation
 - (1) Fundamental Solution.
 - (2) Mean value formulas and Converse to mean -value property.
 - (3) Properties of harmonic functions:
Maximum principle, smoothness, local estimates, Liouville's theorem, analyticity, Harnack's inequality., removable singularity, Schwarz reflection principle.
 - (4) Green's functions for a ball and for a half-space.
 - (5) The classic Dirichlet problem by Perron's method.
 - (6) Newtonian potential and the Hölder estimates for its second derivative.
 - (7) The energy estimates.
- Heat Equation
 - (1) Fundamental Solution.
 - (2) Maximum principle on bounded \bar{U}_T and $\mathbb{R}^n \times [0, T]$ and the uniqueness by maximum principle.
 - (3) Uniqueness and backward uniqueness by energy method.
- Wave Equation
 - (1) Solution by spherical means for dimension 1,2 and 3:
d'Alembert's formula for dimension 1, Kirchhoff's formula for dimension 3 and Poisson's formula for dimension 2.
 - (2) Uniqueness and domain of dependence by energy methods.
- Sobolev Space
 - (1) Definition of Sobolev space.
 - (2) Approximation by smooth functions.
 - (3) Extensions and Traces.

- (4) Sobolev inequalities:
Gagliardo-Nirenberg-Solbolev inequality, Morrey's inequality.
- (5) Compact embedding theorem: Rellich-Kondrachov theorem.
- (6) Poincare's inequality, Difference quotients.

- Second Order Elliptic Equations

- (1) the Definition of weak solutions.
- (2) Existence by Lax-Milgram theorem and energy estimates, by Fredholm alternative.
- (3) Interior Rugularity and boundary regularity.
- (4) Weak and Strong maximum principles, Hopf's lemma.
- (5) Harnack inequality: For the C^2 and $W^{1,2}$ solutions of uniformly elliptic second order equation; For the C_1^2 solutions of uniformly parabolic second order equation, in bounded domains.
- (6) Eigenvalues and eigenfunctions of symmetric elliptic equations
$$Lu = - \sum_{i,j=1}^n (a^{ij}u_{x_i}) x_j.$$

- DeGiorgi Classes

- (1) The definition of DeGiorgi classes.
- (2) Local boundedness and Hölder continuity of functions in Degiogi classes.
- (3) The harnack inequality for functions in Degiogi classes.

Minor Topic: Functional Analysis

- Banach Spaces

- (1) Metric spaces, normed linear vector spaces, Banach spaces and Hahn-Banach theorem.
- (2) Baire category theorem, the Uniform boundedness principle, the open mapping principle and closed graph theorem.
- (3) Dual and reflexive spaces, weak topology and weak* topology.

- Hilbert Space

- (1) Definition of Hilbert space and the dual space of Hilbert space.
- (2) Riesz representation theorem, Lax-Milgram theorem.
- (3) Orthogonal Sets and Basis, Bessel's inequality, Parseval's theorem.

- Compact Operator

- (1) Definition.
- (2) The Riesz-Fredholm theory.

(3) The spectrum of a compact operator.

Reference: Lawrence C. Evans: *Partial Differential Equations*. David Gilbarg . Neil S. Trudinger: *Elliptic Partial Differential Equations of Second Order*. Haim Brezis: *Functional Analysis, Sobolev Spaces and Partial Differential Equations*