

Oral Qualifying Exam Syllabus
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Major Topic: Partial Differential Equations

1. **Laplace's Equation:** fundamental solutions, mean value formulas, properties of harmonic functions, Green's function, Harnack's inequality, Perron's methods
2. **Sobolev Spaces:**
 - (a) weak derivatives: definition of Sobolev spaces
 - (b) approximations: interior/global approximation by smooth functions
 - (c) Extensions, traces, compactness
 - (d) Sobolev inequalities: Gagliardo-Nirenberg-Sobolev inequality, Morrey's inequality, general Sobolev inequalities, Poincaré inequalities
 - (e) Fourier transformation methods
3. **Second order linear elliptic equations:**
 - (a) Existence of weak solutions: Lax-Milgram theorem, energy estimates, Fredholm alternatives
 - (b) Schauder estimate
 - (c) Maximum Principles: weak maximum principle, strong maximum principle, Hopf's lemma, Harnack's inequality
4. **The calculus of Variations:**
 - (a) First variation, Euler-Lagrange equation
 - (b) Existence of minimizers: coercivity, lower semicontinuous, convexity, weak solutions of Euler-Lagrange equations
 - (c) Morse theorem, Mountain Pass theorem
5. **Nonvariational Techniques:**
 - (a) Monotonicity methods
 - (b) Fixed point methods: Banach's fixed point theorem, Schauder's and Schaefer's fixed point theorems
 - (c) Methods of subsolutions and supersolutions

Minor Topic: Algebraic Topology

1. Homotopy theory: covering spaces, lifting criterion, higher homotopy groups, van Kampen Theorem
2. Singular homology theory: definitions, homotopy invariance, relative homology, excision theorem, Mayer-Vietoris sequences, Betti numbers and Euler characteristics
3. Orientation and duality on manifolds: orientation of a manifold, singular cohomology, cup and cap product, Poincaré duality
4. Fibrations: definition, homotopy property, an example—loop spaces
5. CW-complexes: definition and the homology calculation