

Syllabus for Oral Examination

Biao Yin

Comittee: Zheng-chao Han, Yanyan Li, Xiaochun Rong, Richard Wheeden

Elliptic Partial Differential Equations

• Laplace and Poisson Equations

- (a) Mean value properties
- (b) The maximum principle
- (c) Harnack Inequality and Liouville's Theorem
- (d) Fundamental solution of Laplace equation
- (e) Green's representation formula and Poisson integral formula
- (f) Analyticity of harmonic functions
- (g) Perron's method
- (h) Newtonian potential

• Classical Solutions of Second Order Elliptic Equations

- (a) Hopf lemma, Weak/strong maximum principle
- (b) Schauder interior and global estimates
- (c) Existence results by the method of continuity and Fredholm alternative
- (d) Interior and boundary regularity results

• Sobolev Spaces

- (a) Definition of Sobolev spaces
- (b) Extension theorem
- (c) Gagliardo-Nirenberg-Sobolev inequality
- (d) Morrey inequality
- (e) Rellich-Kondrachov compact imbedding theorem
- (f) Poincare inequality
- (g) Difference quotients

- **Weak Solutions of Second Order Elliptic Equations**

- (a) The definition of weak solutions
- (b) Existence results by Lax-Milgram theorem and Fredholm alternative
- (c) Regularity of weak solutions
- (d) Moser iteration, Hanarck inequality
- (e) The calculus of variations: Euler-Lagrange Equation
- (f) Existence of minimizers: coercity, lower semicontinuous, convexity

Riemannian Geometry

- Riemannian metrics
- Levi-Civita connection, Parallel translation
- Geodesics, exponential map, Gauss Lemma
- Normal neighborhood, Convex neighborhood
- Normal coordinates
- Hopf-Rinow theorem
- Curvature tensor
- Sectional curvature, Ricci curvature, Scalar curvature
- Jacobi fields, Conjugate points
- First and second variations of arc length
- Bonnet-Myers theorem
- Cartan-Hadamard theorem
- Rauch comparison theorem
- Space forms
- Differential operators: grad, div and Hess; Divergence theorem

References

- [1] Gilbarg, D and Trudinger, N. S, Elliptic Partial Differential Equations of Second Order, Springer, 1983.
- [2] Evans, L.C. Partial Differential Equations. AMS Providence, 1998
- [3] Cheeger, J and Ebin, G, Comparison Theorems in Riemannian Geometry, North-holland, 1975.
- [4] Carmo, M. P. do, Riemannian Geometry, Birkhauser, Boston, 1992.