# Syllabus for Oral Examination

Biao Yin

Comittee: Zheng-chao Han, Yanyan Li, Xiaochun Rong, Richard Wheeden

## Elliptic Partial Differential Equations

- **Laplace and Poisson Equations**

  (a) Mean value properties
  (b) The maximum principle
  (c) Harnack Inequality and Liouville's Theorem
  (d) Fundamental solution of Laplace equation
  (e) Green's representation formula and Poisson integral formula
  (f) Analyticity of harmonic functions
  (g) Perron's method
  (h) Newtonian potential

- **Classical Solutions of Second Order Elliptic Equations**

  (a) Hopf lemma, Weak/strong maximum principle
  (b) Schauder interior and global estimates
  (c) Existence results by the method of continuity and Fredholm alternative
  (d) Interior and boundary regularity results

- **Sobolev Spaces**

  (a) Definition of Sobolev spaces
  (b) Extension theorem
  (c) Gagliardo-Nirenberg-Sobolev inequality
  (d) Morrey inequality
  (e) Rellich-Kondrachov compact imbedding theorem
  (f) Poincare inequality
  (g) Difference quotients

- **Weak Solutions of Second Order Elliptic Equations**

  (a) The definition of weak solutions
  (b) Existence results by Lax-Milgram theorem and Fredholm alternative
  (c) Regularity of weak solutions
  (d) Moser iteration, Hanarck inequality
  (e) The calculus of variations: Euler-Lagrange Equation
  (f) Existence of minimizers: coercity, lower semicontinuous, convexity

# Riemannian Geometry

- Riemannian metrics
- Levi-Civita connection, Parallel translation
- Geodesics, exponential map, Gauss Lemma
- Normal neighborhood, Convex neighborhood
- Normal coordinates
- Hopf-Rinow theorem
- Curvature tensor
- Sectional curvature, Ricci curvature, Scalar curvature
- Jacobi fields, Conjugate points
- First and second variations of arc length
- Bonnet-Myers theorem
- Cartan-Hadamard theorem
- Rauch comparison theorem
- Space forms
- Differential operators: grad, div and Hess; Divergence theorem

# References


[1] Gilbarg, D and Trudinger, N. S, Elliptic Partial Differential Equations of Second Order, Springer, 1983.

[2] Evans, L.C. Partial Differential Equations. AMS Providence, 1998

[3] Cheeger, J and Ebin, G, Comparison Theorems in Riemannian Geometry, North-holland, 1975.

[4] Carmo, M. P. do, Riemannian Geometry, Birkhauser, Boston, 1992.