Oral Exam Syllabus

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1 Major Topic: Lie Algebras

1. Basic Concepts and Definitions
   (a) Definitions and Standard Examples
   (b) Ideals, Homomorphisms and Representations
   (c) Solvable and Nilpotent Lie Algebras
2. Semisimple Lie Algebras
   (a) Theorems of Lie and Cartan
   (b) Killing Form
   (c) Complete Reducibility of Representations
   (d) Cartan Subalgebras
   (e) Representations of $\mathfrak{sl}_2(\mathbb{F})$
3. Root Systems and Classification
   (a) Axiomatics
   (b) Simple Roots and the Weyl Group
   (c) Classification
   (d) Constructions of Root Systems and Automorphisms

4. Representation Theory of Semisimple Lie Algebras

   (a) Universal Enveloping Algebras
   (b) PBW Theorem
   (c) Serre's Theorem
   (d) Weights and Maximal Vectors
   (e) Finite Dimensional Modules

2 Minor Topic: Trees and Group Actions

1. Graphs, Trees and Morphisms

   (a) Basic Definitions and Examples
   (b) Fundamental Properties of Trees
   (c) Morphisms of Graphs
   (d) Group Actions on Trees

2. Group Actions on Trees

   (a) Quotient Graphs
   (b) Graphs of Groups
   (c) Amalgams and HNN Extensions
   (d) Bass-Serre Structure Theorem

3. Tits Systems and Buildings

   (a) BN-pair Axioms
   (b) Bruhat Decomposition
   (c) Basic Definitions and Properties of Buildings
   (d) Rank 2 and Rank 3 Hyperbolic Buildings

3 References

1. Humphreys, James. *Introduction to Lie Algebras and Representation Theory.* Springer-Verlag, New York, 1972. [Chapters 1, 2, 3, 5].

2. Carbone, Lisa. *Trees and Group Actions.* Yale University Lecture Notes, 1998

3. Brown, Kenneth. *Buildings.* Springer-Verlag, New York, 1989. [Chapter 5]