Topics for oral qualifying exam for Jinwei Yang
Fall, 2009

Major topic: Vertex operator algebras

1. Definitions and properties.
   (a) Formal calculus.
   (b) The notions of vertex algebra and of vertex operator algebra, and basic properties.
   (c) Rationality, commutativity and associativity; equivalence of various formulations, including "weak" formulations.
2. Representations of vertex (operator) algebras.
   (a) The notion of module and basic properties.
   (b) Weak vertex operators.
   (c) The structure of the canonical weak vertex algebra. Local subalgebras and vertex subalgebras of the canonical weak vertex algebra.
   (d) The equivalence between modules and representations.
   (e) General construction theorems for vertex (operator) algebras and modules.
3. Examples of vertex (operator) algebras and modules.
   (a) Vertex (operator) algebras and modules based on the Virasoro algebra.
   (b) Vertex (operator) algebras and modules based on affine Lie algebras.
   (c) Vertex (operator) algebras and modules based on Heisenberg Lie algebras.
   (d) Vertex (operator) algebras and modules on even lattices.
   (e) Vertex operator construction of the affine Lie algebras corresponding to A_n, D_n and E_n.

4. Construction of the Moonshine Module.

   (a) The Golay Code and the Leech Lattice.
   (b) The twisted and untwisted space formed from the Leech Lattice.
   (c) The Moonshine Module and the Griess Algebra.
   (d) Basic properties of the Moonshine Module.

Minor topic: Group Theory

1. Subnormality

   (a) Definition and basic properties
   (b) Nilpotent groups
   (c) Fitting's Theorem
   (d) Frattini subgroup
   (e) Quasisimple and semisimple groups
   (f) Components
   (g) Bender's Theorem

2. Extra-special p-groups

   (a) Definition and basic properties
   (b) Automorphisms of extra-special p-groups

3. Geometry of the classical groups

   (a) Iwasawa's criterion
   (b) The simplicity of the classical groups
   (c) Classification of nondegenerate alternating, sesquilinear, and quadratic forms over finite fields
   (d) Witt's Lemma
   (e) Buildings and BN pairs associated to the classical groups

References

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