

Syllabus for Oral Examination

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Partial Differential Equations

- **Laplace's and Heat equations**

1. **Laplace's and Poisson's equation:** fundamental solutions, mean value property, the maximum principle, Green's representation formula and Poisson integral formula, Harnack's inequality and Liouville's theorem, analyticity, Perron's methods
2. **Heat equation:** solution to initial-value problem, mean value property, the maximum principle, estimates on derivatives, energy estimate and backward uniqueness

- **Classical Solutions of Second Order Elliptic Equations**

- (a) Hopf lemma, Weak/strong maximum principle
- (b) Newtonian potential
- (c) Schauder interior and global estimates
- (d) Existence results by the method of continuity and Fredholm alternative

- **Sobolev Spaces**

- (a) Definition of Sobolev spaces
- (b) Meyers-Serrin theorem
- (c) Extension theorem
- (d) Gagliardo-Nirenberg-Sobolev inequality
- (e) Morrey inequality
- (f) Rellich-Kondrachov compact imbedding theorem
- (g) Poincaré inequality
- (h) Difference quotients
- (i) Spaces involving time

- **Weak Solutions of Second Order Elliptic and Parabolic Equations**

1. **elliptic equation:** Definition of weak solutions, Existence results by Lax-Milgram theorem and Fredholm alternative, Regularity of weak solutions, Eigenvalue and Eigenfunction of symmetric elliptic operator, De Giorgi estimates and Global Boundedness
2. **Parabolic equation:** Definition of weak solution, energy estimates, Galerkin approximation, Existence and uniqueness

Riemannian Geometry

- Riemannian metrics
- Levi-Civita connection, Parallel translation
- Exponential map, Gauss Lemma
- Geodesics, Isometry
- Sectional curvature, Ricci curvature, Scalar curvature
- Jacobi fields, Conjugate points
- Manifold and map
- Vector bundles and Curvature tensor
- Hopf-Rinow theorem, Completeness
- First and second variations of arc length
- Bonnet-Myers theorem
- Cartan-Hadamard theorem
- Connections and Differential forms
- Space forms
- Differential operators: grad, div, and Hess; Divergence theorem

References

- [1] Gilbarg, D and Trudinger, N. S, Elliptic Partial Differential Equations of Second Order, Springer, 1983.
- [2] Evans, L.C. Partial Differential Equations. AMS Providence, 1998
- [3] Grove Karsten, Riemannian Geometry: A metric Entrance, Lecture notes, U Aharhus, 1999.
- [4] Jost, J, Riemannian geometry and Geometric analysis, Springer, 2002.