Syllabus for Oral Qualifying Exam

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I. **Several Complex Variables**

1. Cauchy integral formula and its applications:

- Cauchy integral formula in polydiscs, Cauchy estimates;
- Bochner-Martinelli formula;
- Hartogs extension Theorem;
- Bochner's extension Theorem.

2. Subharmonicity and convexity:

- Properties of subharmonic functions and plurisubharmonic functions;
- Domain of holomorphy, Continuity Principle;
- Pseudoconvexity, Levi pseudoconvexity;
- Oka's Theorem.

3. L^2 theory for $\bar{\partial}$ on pseudoconvex domains:

- $\bar{\partial}$-Neumann problem;
- Morrey-Kohn-Hormander Theorem;
- L^2 Existence Theorem for $\bar{\partial}$ operator;
- L^2 Existence Theorem for $\bar{\partial}$-Neumann operator;
- The Levi problem;
- Subelliptic estimates for $\bar{\partial}$-Neumann operator.

4. Tangential Cauchy-Riemann Complex:

- Definition of CR manifold;
- Subelliptic estimates for the Tangential Cauchy-Riemann Complex.

II. **Partial Differential Equations**

1. Fourier analysis and Sobolev spaces:

- Fourier transform;
- Definition of Sobolev space;
- Density Theorem of smooth functions into Sobolev space;
- Sobolev embedding Theorem;
- Rellich embedding Theorem.

2. Second order elliptic equations:

- Weak maximum principle for classical second order elliptic equations;
- Hopf Lemma;
- Definition of weak solution;
- Lax-Milgram Theorem;
- Existence of weak solution;
- Regularity of weak solution.

3. Subelliptic analysis:

- Definition of pseudo-differential operator;
- Hormander's hypoelliptic Theorem.

References

[CS] So-chin Chen, Mei-chi Shaw, Partial Differential Equations in Several Complex Variables.

[Kr] Steven G. Krantz, Function Theory of Several Complex Variables.

[Oh] Takeo Ohsawa, Analysis of Several Complex Variables.

[Ev] Evans, Lawrence C, Partial Differential Equations.