

GEOMETRY AND PHYSICS

Essays in honour of
I.M. Gelfand

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ISRAEL GELFAND

I. M. Gelfand's research work is well known, although of course many of its aspects are worth discussing and studying. The circumstances of his life are not known so widely; I feel that they are also of interest to the mathematical community. Perhaps Gelfand's 75th anniversary is an appropriate occasion to recall a few milestones of his life story.

Israel Moiseevich Gelfand was born on September 2nd, 1913 in the *mestechko* (= shtetl = small town) Krasnye Okna near Odessa. The expressive word *mestechko* has now entirely disappeared from Russian geographical terminology, although the expression *mestechkovyi evrei* (= shtetl jew) is still used in the Russian language when the speaker wishes to stress his irritation with the person in question. The size of the *mestechko* may be judged by the fact that there was only one school there at the time. The math teacher, however, was excellent (one of Gelfand's class mates – David Milman – also became a research mathematician).

Israel Gelfand stayed in Krasnye Okna until he was almost 17 and, in the last 5 years there, lived an active mathematical life, in almost total isolation from mainstream mathematics, recreating a mathematical world of his own on the basis of the scarce information available to him. I. M. Gelfand willingly recalls this period of his life. Recently his life story appeared in *Kvant*, the Soviet popular science magazine for high school students. Hopefully, it will soon become available to the Western reader.

I. M. Gelfand believes that these five years (1925-1929) completely determined his «way of doing mathematics», his «artistic image» of the subject. He tends to assess as positive the facts that he did not get any college education at all and that his road to science was far from canonical. At his disposal in Krasnye Okna he had only a few secondary school text books and two or three more advanced books, acquired by accident. He studied the differential calculus using a technical school textbook, written in Ukrainian by someone called Belyaev. To give an idea of its scientific level, Gelfand likes to say that it begins with an explanation of what functions are by stating that there are three kinds of functions: analytic (those given by formulas), empirical (given by tables) and «correlational». This mysterious third class of functions was not described in any way and remained a puzzle for Gelfand for many years. He got his hands on the book at the age of 15. Under its influence he reunified mathematics in his own consciousness into a single discipline from its two varieties – the algebraic and geometric (= transcendental) ones. The source of this profound change of outlook were infinite series, which may be used to express analytically functions of geometric origin (e.g. trigonometric ones). There are many well known examples from Euler to Ramanujan – of the overwhelming impression that infinite series produce on the youthful inquisitive mind. A deep conviction of the unity of all mathematics – an important trait of I. M. Gelfand's scientific outlook – stems from that period. A detailed description of specific problems tackled by Gelfand at that time (and his solutions), which would be out of place here, only confirms how much of Gelfand's style was determined by those formative years.

1929 – the «year of the great breakthrough» in Russian history did indeed break more than one life. Numerous people fled to the largest cities to save themselves from hunger, unemployment, repression by local authorities. Gelfand, not yet 17, leaves his parents for Moscow, where he is housed by relatives. He is unsuccessful in his attempts to find a steady job and this leaves him time for regular visits to the National (Lenin) Library. A 45 minute walk (no money to spend on carfare!) takes him from Pokrovka, where his relatives live, to the sanctuary of the Library, where any books or journals are available (today, the absence of «appropriate» documents which include a university diploma, unfailingly keeps potential Gelfands out of that institution), where one can find people with similar interests, mathematics students capable of answering questions and a cafeteria where 15 kopecks suffice for a frugal meal-tea, bread, Russian salad. For a while Gelfand has the good fortune to get a job as a library controller, but a change in the management results in his losing that job: as was often to happen in the future, I. M. Gelfand's looks are deemed «unphotogenic» (his own expression) by his superior. He is fired.

An avalanche of new information about modern (1929) mathematics pours down on the 17-year old Gelfand: he finds out that his «discoveries» are not new. As he says to himself, «the pure experiment ends». He finds it very difficult to find his way in the suddenly widened field of mathematics. Not long before his arrival to Moscow, he had

decided that an «arbitrary function» is any sum of a convergent power series (he was proving that any even function of a real variable viewed as a function of complex variable assumes only real values on the imaginary axis, as does $\cos ix$, with which he began his considerations). In Moscow, however, the «in» mathematics then was the theory of a function of real variable, and Gelfand learned all about the tricks related to the notion of function. To overcome what he now sees as an old-fashioned attitude, he works his way through Vallee-Poussin's treatise, but only the study of analytic (complex) functions via Privalov's textbook restores his feeling of harmony in the mathematical world.

By the age of 18, Gelfand has the possibility of doing mathematics professionally and he seeks a teaching position. The situation in this respect turns out to be unexpectedly favorable. The country is intent on creating a new technical and scientific intelligentsia from rank and file industrial workers. In this question, as in many others, the general attitude is that positive results can be achieved very quickly. This leads to the introduction of educational methods of various degrees of thoughtlessness, new selfmade institutions of learning with pompous names multiply. Gelfand becomes a consultant in one of them – the Bubnov Higher Engineering School. Attempts to put some order into the system begin by 1931, although the range of institutions is not narrowed and professional teachers are scarce. Thus in the Evening Chemico-Technological Institute more than 30 groups (working by the «brigade-laboratory» method) are working. Gelfand works as a «stand by instructor», replacing absent teachers, in fact, almost every day. But here also he does not succeed in producing a good impression on his «superiors» and this job, which he had found attractive, is quickly terminated.

By the end of 1931 Gelfand becomes a habitué of Moscow University seminars, observes *bonafide* students (among them – M. V. Keldysh, A. I. Maltsev), becomes more and more active in University life. In 1932 he becomes a post-graduate student, without any college or even high school diploma! Remarkable things were possible in those terrible times. This story became the basis of well known legend: a university math professor, having found that the foreign journal he needed had been taken out by a young library controller, had a short talk with him and invited him to do post graduate work at Moscow University. Be that as it may, but one thing can be said for sure: today this could not possibly happen.

From then on Gelfand's life runs fairly smoothly: in 1935 he finishes his graduate work, defends his «candidate's» thesis (Ph.D. level), then his doctorate (1940), from 1933 teaches at Moscow University, in 1943 becomes full professor (in 1952 he loses his professor's title for a while, but he is allowed to continue conducting his university seminar), for many years works at the Steklov Mathematics Institute of the Academy of Sciences, then at the Institute for Applied Mathematics. In 1953 Gelfand is elected corresponding member of the Academy of Sciences, but only 30 years later becomes a full fledged member (looking at things from the outside, it seems that this could have happened earlier!). By then I. M. Gelfand was a member of numerous foreign academies,

doctor *honoris causa* of several prestigious universities.

I. M. Gelfand has been working in mathematics, for over 50 years. Normed rings (Banach algebras), representations of semi-simple Lie groups, the Sturm-Liouville inverse problem, integral geometry, homology of infinite dimensional algebras, formal geometry, the general theory of hypergeometric functions – are only a few milestones in this remarkable life of scientific research. At present, *Springer Verlag* is concluding its work on the edition of his *Selecta*. Three volumes, more than 3.000 pages could not cover all that he has written. The *Selecta* do not include any of the books, e.g. «Generalized functions» (= «Distributions») in 6 volumes, many articles. I was one of the editors, and in the process of choosing the contents of the volumes was amazed by the rigorous inner logic followed by the author in his choice of research topics.

I. M. Gelfand plays an important role in the mathematical life in Moscow. For more than 45 years he has headed the famous seminar in functional analysis usually known by his name. At various times almost all Moscow mathematicians were its participants, and his influence was considerable not only on his direct pupils. Since 1965 he is the Editor-in-Chief of the Journal *Functional analysis and its Applications*, which he founded.

I. M. Gelfand has reached his 75th anniversary without cutting down on his research activity, full of new plans and ideas. He is preoccupied with the creation of the general theory of hyper-geometric functions (this topic can be traced back to earlier work in representation theory and the theory of distributions), but other endeavors are beginning to be carried out: multidimensional determinants, combinatorics, integral geometry revisited ...

S. Gindikin