

Tribute to I. M. Gelfand for his 80th Birthday Celebration

I. M. Singer

We are here to honor Israel Gelfand and to celebrate the continued vitality of one of the most influential mathematicians of the twentieth century—I dare say, the most outstanding of the last fifty years.

Unfortunately, our society neither understands nor appreciates mathematics. Despite its many applications, despite its intellectual power which has changed the way we do science, mathematicians are undervalued and ignored.

Naturally, its practitioners, its leaders, go unrecognized. They have neither power nor influence. Watching the negative effects popularity causes in other fields, and wincing at the few superficial articles about mathematics, I think it is just as well.

Faced constantly with problems we can't solve, most mathematicians tend to be modest about themselves and their accomplishments. Perhaps that is why we have failed to recognize a giant in our midst. I won't compare Gelfand with other outstanding mathematicians or scientists of the twentieth century; if I did, you would stop listening and start checking for yourselves whether you agree with me. But focus on my point—*we have a giant in our midst*. I turn to other fields to find comparable achievements: Balanchine in dance, or Thomas Mann in literature, or Stravinsky, better still, Mozart in music; but for me, a better comparison is with artists like Cézanne and Matisse. I commend to you the great poet Paul Rilke's letter on Cézanne. He said, "Paul Cézanne has been my supreme example, because he has remained in the innermost center of his work for forty years... which explains something beyond the freshness and purity of his paintings" (of course, for Gelfand, 60 years).

Evoking Matisse is perhaps more apt. A Matisse is breathtaking. No matter what his personal circumstance, he turns to new frontiers with joy and energy. Particularly outstanding is his later work: Jazz, and the remarkable "papier-découpés"—efforts done in his early eighties.

Gelfand too continues to dazzle us with new and profound ideas. His latest book with Kapranov and Zelevinsky is a major work that maps out new directions for decades to come.

In preparing this tribute, I asked many people for topics I should emphasize today. You will be interested in what happened. First, there was little intersection in the subjects my correspondents chose. Second, everyone gave me a five to twenty minute enthusiastic lecture on the essence of Gelfand's contribution—simple, and profound.

Reviewing Gelfand's contributions to mathematics has been an education.

Let me remind you of some of his main work.

1. Normed Rings
2. C^* -Algebras (with Raikov)—the GNS Construction
3. Representations of complex and real semi-simple groups
(with Neumark and Graev)
4. Integral Geometry— Generalizations of the Radon Transform
5. Inverse scattering of Sturm Liouville systems (with Levitan)
6. Gelfand-Dickey on Lax operators and KdV
7. The treatises on generalized functions
8. On elliptic equations
9. The cohomology of infinite dimensional Lie algebras (with Fuks)
10. Combinatorial characteristic classes (beginning with MacPherson)
11. Dilogarithms, discriminants, hypergeometric functions
12. The Gelfand Seminar

It is impossible to review his enormous contributions in a few minutes. If I were Gelfand himself, I would orchestrate this occasion, like his seminar, by calling on many of you unexpectedly and demanding a one-sentence synopsis of a particular paper. But rather than intimidate you, I will comment on a few results that affected me.

As a graduate student, one of the first strong influences on me was Gelfand's Normed Ring paper. Marshall Stone had already taught us that points could be recaptured in Boolean algebras as maximal ideals. But Gelfand combined analysis with algebra in a simple and beautiful way. Using maximal ideals in a complex commutative Banach algebra, he represented such algebras as algebras of functions. Thus began the theory of commutative Banach algebras. The spectral theorem and the Wiener Tauberian Theorem were elementary consequences. I was greatly influenced by the revolutionary view begun there.

A natural next step for Gelfand was the study of non-commutative C^* -algebras. He represented such algebras as operator algebras using the famous GNS construction. It seemed inevitable to find unitary representations of locally compact groups using their convolution algebras. The representation theory of complex and real semi-simple Lie groups followed quickly after. What struck me most was the geometric approach Gelfand and his coworkers took. Only recently, it appears this subject has become geometric again.

In 1963, twenty American experts in PDEs were on their way to Novosibirsk for the first visit of foreign scientists to the academic city there. It was in the midst of a Khrushchev thaw. When I learned about it, I asked whether I could be added to the list of visitors, citing the index theorem Atiyah and I had just proved. After reading his early papers, I wanted to meet Gelfand. Each day of my two week stay in Novosibirsk I asked Gelfand's students when he was coming. The response was always "tomorrow." Gelfand never came. I sadly returned to Moscow. When I got to my room at the infamous Hotel Ukraine,

the telephone rang and someone said Gelfand wanted to meet me; could I come downstairs. There was Gelfand. He invited Peter Lax and me for a walk. During the walk, Peter tried to tell Gelfand about his work on $SL(2, R)$ with Ralph Phillips. Gelfand tried to explain his own view of $SL(2, R)$ to Peter, but his English was inadequate. (He was rusty; within two days his English was fluent.) I interrupted and explained Gelfand's program to Peter. At the corner Gelfand stopped, turned to me, and said: "*But you are my student.*" I replied, "Indeed, I *am* your student." (By the way, Gelfand told me he didn't come to Novosibirsk because he hates long conferences. That's why this celebration lasts only four days.)

Although it is an honor to be a Gelfand student, it is also a burden. We try to imitate the depth and unity that Gelfand brings to mathematics. He makes us think harder than we believed possible. Gelfand and I became close friends in a matter of minutes, and have remained so ever since. I was ill in Moscow, and Gelfand took care of me.

I didn't see him again for ten years. He was scheduled to receive an honorary degree at Oxford, where I was visiting. It was unclear that he would be allowed to leave the Soviet Union to visit the West. I decided not to wait and returned home. A week later, I received a telegram from Atiyah; Gelfand was coming—the Queen had asked the Russian ambassador to intercede. I flew back to England and accompanied Gelfand during his visit, a glorious time. Many things stood out.

But I'll mention only one, our visit to a Parker Fountain Pen store. Those of you who have ever shopped with Gelfand are smiling; it is always an unforgettable experience. Within fifteen minutes, he had every salesperson scrambling for different pens. Within an hour, I knew more about the construction of fountain pens than I ever cared to know, and had ever believed possible! Gelfand's infinite curiosity and the focused energy on details are unbelievable; that, coupled with his profound intuition of essential features is rare among human beings. He is beyond category.

Talking about Oxford, let me emphasize Gelfand's paper on elliptic equations. In 1962, Atiyah and I had found the Dirac operator on spin manifolds and already had the index formula for geometric operators coupled to any vector bundle, although it took another nine months to prove our theorem. Gelfand's paper was brought to our attention by Smale. It enlarged our view considerably, as Gelfand always does, and we quickly realized, using essentially the Bott periodicity theorem, that we could prove the index theorem for any elliptic operator.

I haven't talked about the applications of Gelfand's work to Physics—Gelfand-Fuks, for example, on vector fields of the circle, the so-called Virasoro Algebra, which Virasoro did not in fact define. Although I mentioned Gelfand-Dickey, I haven't stressed its influence very recently on matrix model theory. Nor have I described how encouraging he is and how far ahead of his time he is

in understanding the implications of a paper which seems obscure at the time.

Claude Itzykson told me that his now famous paper with Brezin, Parisi and Zuber that led to present-day methods of triangulating moduli space went unnoticed by scientists. The authors received one request for a reprint—from Gelfand.

Ray and I were very excited about our definition of determinants for Laplacian-like operators and its use in obtaining manifold invariants-analytic torsion. The early response in the U.S. was silence; Gelfand sent us a congratulatory telegram.

It has been a great honor to have been chosen to pay tribute to Gelfand on this very special occasion. As you can tell, he means a great deal to me personally.

Among his many special qualities, I will mention only one in closing. He is a magician. It is not very difficult, not very difficult at all, for any of us mere mortals to keep the difference in our ages a constant function of time. But with Gelfand... when I met him 30 years ago, and 20 years ago, I thought Gelfand was older than I. About ten years ago, I felt we were the same age. Now it is quite clear that he is younger; in fact, *much* younger than most in the audience. It is important for us all that Gelfand continue to prosper and to do such great mathematics.

We wish him good health and happiness.

I. M. Singer