

ALGEBRA I, FALL 2015, HOMEWORK 1

**Due:** Thursday, September 24, in class.

**Problems:** (All from Basic Algebra I.)

**1.2:** 7, 13

**1.4:** 3

Hint: Consider the permutation group  $S(A) \cong S_9$  of the finite abelian group  $A = \mathbb{Z}/3\mathbb{Z} \oplus \mathbb{Z}/3\mathbb{Z}$ . Let  $\alpha \in S(A)$  be defined by  $\alpha(x, y) = (y, -x - y)$ . Let  $G \leq S(A)$  be the subgroup generated by  $\alpha$  and all translations by elements of  $A$ .

**1.6:** 2

**1.7:** 4

Hint: Choose finite subsets  $S, T \subset G$  such that  $G = \langle S \rangle$  and  $G = TH$ . Consider the finite set  $U = (TST^{-1} \cup TS^{-1}T^{-1} \cup T \cup T^{-1}) \cap H$ , and let  $\langle U \rangle \leq H$  be the subgroup generated by  $U$ . Show first that  $G = T\langle U \rangle$ , then that  $H = \langle U \rangle$ .

**1.8:** 11

Hint: Consider the subgroup of all elements  $g \in G$  for which the left translation  $g_L : G \rightarrow G$  is an even permutation of  $G$ .

**1.9:** 1

STAPLE YOUR WORK!!!