GEOMETRY 435 FINAL EXAM FORMULA SHEET

Ellipse in standard form.

An ellipse in standard form has equation $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ where $a \geq b > 0$.
Eccentricity: $e = \sqrt{1 - \frac{b^2}{a^2}}$
Foci: $(\pm ae, 0)$
Directrices: $x = \pm \frac{a}{e}$

Parabola in standard form.

A parabola in standard form has equation $y^2 = 4ax$ where $a > 0$.
Eccentricity: $e = 1$
Focus: $(a, 0)$
Directrix: $x = -a$

Hyperbola in standard form.

A hyperbola in standard form has equation $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ where $a, b > 0$.
Eccentricity: $e = \sqrt{1 + \frac{b^2}{a^2}}$
Foci: $(\pm ae, 0)$
Directrices: $x = \pm \frac{a}{e}$.

Cross Ratio in $\mathbb{RP}^2$.

Let $A = [\overrightarrow{a}]$, $B = [\overrightarrow{b}]$, $C = [\overrightarrow{c}]$, $D = [\overrightarrow{d}]$ be distinct colinear points in $\mathbb{RP}^2$.
Then $(ABCD) = \dfrac{\beta/\alpha}{\delta/\gamma}$, where $\overrightarrow{c} = \alpha\overrightarrow{a} + \beta\overrightarrow{b}$ and $\overrightarrow{d} = \gamma\overrightarrow{a} + \delta\overrightarrow{b}$.

Cross ration in $\mathbb{R}^2$.

Let $A, B, C, D \in \mathbb{R}^2$ be distinct colinear points.
Then $(ABCD) = \dfrac{AC \cdot BD}{AD \cdot BC}$.

Polars and tangent pairs.

Let $E \subset \mathbb{RP}^2$ be the non-degenerate conic with equation $s = 0$,
$s = Ax^2 + Bxy + Cy^2 + Fxz + Gyz + Hz^2$.
Given $P_1 = [x_1, y_1, z_1] \in \mathbb{RP}^2$, set
$s_1 = Ax_1x + B\dfrac{x_1y + y_1x}{2} + Cy_1y + F\dfrac{x_1z + z_1x}{2} + G\dfrac{y_1z + z_1y}{2} + Hz_1z$.
Polar of P_1 wrt. $E : s_1 = 0$.
Tangent pair to E from $P_1 : (s_1)^2 - s(x_1, y_1, z_1)\, s = 0$.

Equation for conic.

The non-degenerate conic $E \subset \mathbb{RP}^2$ through the points
$[1, 0, 0]$, $[0, 1, 0]$, $[0, 0, 1]$, $[1, 1, 1]$, $[a, b, c]$ has equation
$c(a - b)xy + a(b - c)yz + b(c - a)zx = 0$.

Parametrization.

Every point other than $[1, 0, 0]$ on the projective conic $xy + yz + zx = 0$
can be expressed as $[t^2 + t, t + 1, -t]$ for some $t \in \mathbb{R}$.