GOODIES 3

Problem 1. Prove that the Segre map $s : \mathbb{P}^n \times \mathbb{P}^m \to \mathbb{P}^N$ gives an isomorphism of $\mathbb{P}^n \times \mathbb{P}^m$ with a closed subvariety of $\mathbb{P}^N$, where $N = nm + n + m$.

Problem 2. Let E be the elliptic curve $V_+(y^2z - x^3 + xz^2) \subset \mathbb{P}^2$ and let $f, g : E \dashrightarrow \mathbb{P}^1$ be the rational maps defined by $f(x : y : z) = (x : z)$ and $g(x : y : z) = (y : z)$. (These are just projections to the x and y axis on the open subset $D_+(z)$.)

(a) Find the maximal open sets in E where f and g are defined as morphisms.

(b) Find the degrees of the field extensions $k(t) \subset k(E)$ induced by f and g.

(c) Find the cardinality of $f^{-1}(p)$ and $g^{-1}(p)$ when $p \in \mathbb{P}^1$ is a typical point. (Part of the exercise is to define what "typical" means.)

Problem 3. Let X be a projective variety and $\varphi : \mathbb{P}^1 \dashrightarrow X$ any rational map. Show that φ is defined as a morphism on all of $\mathbb{P}^1$.

Problem 4. Let $E = V(y^2 - x^3 + x) \subset \mathbb{A}^2$. Show that if $P \in E$ is any point then $E \smallsetminus \{P\}$ is affine.