Integer Problem:

Maximize $z = 6x_1 + 7x_2$
subject to

$6x_1 + 8x_2 \ \leq \ 16$

$(x_1, x_2) \geq 0$ in $\mathbb{Z}^2$

Canonical Form:

Maximize $z = 6x_1 + 7x_2$
subject to

$6x_1 + 8x_2 + u_1 \;=\; 16$

$(x_1, x_2, u_1) \geq 0$ in $\mathbb{Z}^3$

	x_1	x_2	u_1	
u_1	6	8	1	16
	-6	-7	0	0

	x_1	x_2	u_1	
u_1	6	8	1	16
	-6	-7	0	0

	x_1	x_2	u_1	
← u_1	6	8	1	16
	-6	-7	0	0

x_1	x_2	u_1	
1	4/3	1/6	8/3
-6	-7	0	0

	x_1	x_2	u_1	
x_1	1	4/3	1/6	8/3
	0	1	1	16

	x_1	x_2	u_1	
x_1	1	4/3	1/6	8/3
	0	1	1	16

Root of tree:

$z = 16$
$x_1 = 8/3 = 2\frac{2}{3}$
$x_2 = 0$

Branch the leaf:

$z = 16$
$x_1 = 8/3 = 2\frac{2}{3}$
$x_2 = 0$

New constraints: $x_1 \leq 2$ and $x_1 \geq 3$

	x_1	x_2	u_1	
x_1	1	4/3	1/6	8/3
	0	1	1	16

Add constraint: $x_1 \leq 2$

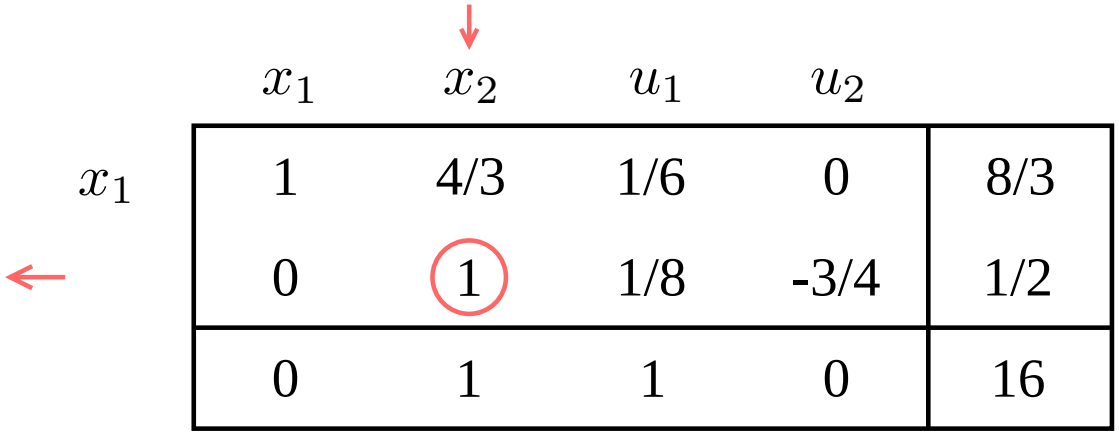
	x_1	x_2	u_1	u_2	
x_1	1	4/3	1/6	0	8/3
u_2	1	0	0	1	2
	0	1	1	0	16

	x_1	x_2	u_1	u_2	
x_1	1	4/3	1/6	0	8/3
u_2	1	0	0	1	2
	0	1	1	0	16

	x_1	x_2	u_1	u_2	
x_1	1	4/3	1/6	0	8/3
u_2	0	-4/3	-1/6	1	-2/3
	0	1	1	0	16

	x_1	x_2	u_1	u_2	
x_1	1	4/3	1/6	0	8/3
← u_2	0	-4/3	-1/6	1	-2/3
	0	1	1	0	16

	x_1	x_2	u_1	u_2	
x_1	1	4/3	1/6	0	8/3
← u_2	0	-4/3	-1/6	1	-2/3
	0	1	1	0	16


x_1 x_2 u_1 u_2
x_1
1 4/3 1/6 0 8/3
0 1 1/8 -3/4 1/2
0 1 1 0 16

	x_1	x_2	u_1	u_2	
x_1	1	0	0	1	2
x_2	0	1	1/8	-3/4	1/2
	0	0	7/8	3/4	31/2

	x_1	x_2	u_1	u_2	
x_1	1	0	0	1	2
x_2	0	1	1/8	-3/4	1/2
	0	0	7/8	3/4	31/2

Left child:

$z = 31/2 = 15\frac{1}{2}$
$x_1 = 2$
$x_2 = 1/2 = \frac{1}{2}$

	x_1	x_2	u_1	
x_1	1	4/3	1/6	8/3
	0	1	1	16

Add constraint: $x_1 \geq 3$

	x_1	x_2	u_1	u_2	
x_1	1	4/3	1/6	0	8/3
u_2	-1	0	0	1	-3
	0	1	1	0	16

	x_1	x_2	u_1	u_2	
x_1	1	4/3	1/6	0	8/3
u_2	-1	0	0	1	-3
	0	1	1	0	16

	x_1	x_2	u_1	u_2	
x_1	1	4/3	1/6	0	8/3
u_2	0	4/3	1/6	1	-1/3
	0	1	1	0	16

	x_1	x_2	u_1	u_2	
x_1	1	4/3	1/6	0	8/3
u_2	0	4/3	1/6	1	-1/3
	0	1	1	0	16

Right child: $\emptyset$

Branch the leaf:

$z = 31/2 = 15\frac{1}{2}$
$x_1 = 2$
$x_2 = 1/2 = \frac{1}{2}$

New constraints: $x_2 \leq 0$ and $x_2 \geq 1$

	x_1	x_2	u_1	u_2	
x_1	1	0	0	1	2
x_2	0	1	1/8	-3/4	1/2
	0	0	7/8	3/4	31/2

Add constraint: $x_2 \leq 0$

	x_1	x_2	u_1	u_2	u_3	
x_1	1	0	0	1	0	2
x_2	0	1	1/8	-3/4	0	1/2
u_3	0	1	0	0	1	0
	0	0	7/8	3/4	0	31/2

	x_1	x_2	u_1	u_2	u_3	
x_1	1	0	0	1	0	2
x_2	0	1	1/8	-3/4	0	1/2
u_3	0	1	0	0	1	0
	0	0	7/8	3/4	0	31/2

	x_1	x_2	u_1	u_2	u_3	
x_1	1	0	0	1	0	2
x_2	0	1	1/8	-3/4	0	1/2
u_3	0	0	-1/8	3/4	1	-1/2
	0	0	7/8	3/4	0	31/2

	x_1	x_2	u_1	u_2	u_3	
x_1	1	0	0	1	0	2
x_2	0	1	1/8	-3/4	0	1/2
← u_3	0	0	-1/8	3/4	1	-1/2
	0	0	7/8	3/4	0	31/2

	x_1	x_2	u_1	u_2	u_3	
x_1	1	0	0	1	0	2
x_2	0	1	1/8	-3/4	0	1/2
u_3	0	0	-1/8	3/4	1	-1/2
	0	0	7/8	3/4	0	31/2


x_1 x_2 u_1 u_2 u_3
x_1 1 0 0 1 0 2
x_2 0 1 1/8 -3/4 0 1/2
0 0 1 -6 -8 4
0 0 7/8 3/4 0 31/2

	x_1	x_2	u_1	u_2	u_3	
x_1	1	0	0	1	0	2
x_2	0	1	0	0	1	0
u_1	0	0	1	-6	-8	4
	0	0	0	6	7	12

	x_1	x_2	u_1	u_2	u_3	
x_1	1	0	0	1	0	2
x_2	0	1	0	0	1	0
u_1	0	0	1	-6	-8	4
	0	0	0	6	7	12

Left child:

$z = 12$
$x_1 = 2$ $x_2 = 0$

	x_1	x_2	u_1	u_2	
x_1	1	0	0	1	2
x_2	0	1	1/8	-3/4	1/2
	0	0	7/8	3/4	31/2

Add constraint: $x_2 \geq 1$

	x_1	x_2	u_1	u_2	u_3	
x_1	1	0	0	1	0	2
x_2	0	1	1/8	-3/4	0	1/2
u_3	0	-1	0	0	1	-1
	0	0	7/8	3/4	0	31/2

	x_1	x_2	u_1	u_2	u_3	
x_1	1	0	0	1	0	2
x_2	0	1	1/8	-3/4	0	1/2
u_3	0	-1	0	0	1	-1
	0	0	7/8	3/4	0	31/2

	x_1	x_2	u_1	u_2	u_3	
x_1	1	0	0	1	0	2
x_2	0	1	1/8	-3/4	0	1/2
u_3	0	0	1/8	-3/4	1	-1/2
	0	0	7/8	3/4	0	31/2

	x_1	x_2	u_1	u_2	u_3	
x_1	1	0	0	1	0	2
x_2	0	1	1/8	-3/4	0	1/2
← u_3	0	0	1/8	-3/4	1	-1/2
	0	0	7/8	3/4	0	31/2

	x_1	x_2	u_1	u_2	u_3	
x_1	1	0	0	1	0	2
x_2	0	1	1/8	-3/4	0	1/2
u_3	0	0	1/8	-3/4	1	-1/2
	0	0	7/8	3/4	0	31/2

	x_1	x_2	u_1	u_2	u_3	
x_1	1	0	0	1	0	2
x_2	0	1	1/8	-3/4	0	1/2
←	0	0	-1/6	1	-4/3	2/3
	0	0	7/8	3/4	0	31/2

	x_1	x_2	u_1	u_2	u_3	
x_1	1	0	1/6	0	4/3	4/3
x_2	0	1	0	0	-1	1
u_2	0	0	-1/6	1	-4/3	2/3
	0	0	1	0	1	15

	x_1	x_2	u_1	u_2	u_3	
x_1	1	0	1/6	0	4/3	4/3
x_2	0	1	0	0	-1	1
u_2	0	0	-1/6	1	-4/3	2/3
	0	0	1	0	1	15

Right child:

$z = 15$
$x_1 = 4/3 = 1\frac{1}{3}$
$x_2 = 1$

Branch the leaf:

$z = 15$
$x_1 = 4/3 = 1\frac{1}{3}$
$x_2 = 1$

New constraints: $x_1 \leq 1$ and $x_1 \geq 2$

	x_1	x_2	u_1	u_2	u_3	
x_1	1	0	1/6	0	4/3	4/3
x_2	0	1	0	0	-1	1
u_2	0	0	-1/6	1	-4/3	2/3
	0	0	1	0	1	15

Add constraint: $x_1 \leq 1$

	x_1	x_2	u_1	u_2	u_3	u_4	
x_1	1	0	1/6	0	4/3	0	4/3
x_2	0	1	0	0	-1	0	1
u_2	0	0	-1/6	1	-4/3	0	2/3
u_4	1	0	0	0	0	1	1
	0	0	1	0	1	0	15

	x_1	x_2	u_1	u_2	u_3	u_4	
x_1	1	0	1/6	0	4/3	0	4/3
x_2	0	1	0	0	-1	0	1
u_2	0	0	-1/6	1	-4/3	0	2/3
u_4	1	0	0	0	0	1	1
	0	0	1	0	1	0	15

	x_1	x_2	u_1	u_2	u_3	u_4	
x_1	1	0	1/6	0	4/3	0	4/3
x_2	0	1	0	0	-1	0	1
u_2	0	0	-1/6	1	-4/3	0	2/3
u_4	0	0	-1/6	0	-4/3	1	-1/3
	0	0	1	0	1	0	15

	x_1	x_2	u_1	u_2	u_3	u_4	
x_1	1	0	1/6	0	4/3	0	4/3
x_2	0	1	0	0	-1	0	1
u_2	0	0	-1/6	1	-4/3	0	2/3
← u_4	0	0	-1/6	0	-4/3	1	-1/3
	0	0	1	0	1	0	15

	x_1	x_2	u_1	u_2	u_3 ↓	u_4	
x_1	1	0	1/6	0	4/3	0	4/3
x_2	0	1	0	0	-1	0	1
u_2	0	0	-1/6	1	-4/3	0	2/3
← u_4	0	0	-1/6	0	-4/3	1	-1/3
	0	0	1	0	1	0	15

	x_1	x_2	u_1	u_2	u_3	u_4	
x_1	1	0	1/6	0	4/3	0	4/3
x_2	0	1	0	0	-1	0	1
u_2	0	0	-1/6	1	-4/3	0	2/3
←	0	0	1/8	0	1	-3/4	1/4
	0	0	1	0	1	0	15

	x_1	x_2	u_1	u_2	u_3	u_4	
x_1	1	0	0	0	0	1	1
x_2	0	1	1/8	0	0	-3/4	5/4
u_2	0	0	0	1	0	-1	1
u_3	0	0	1/8	0	1	-3/4	1/4
	0	0	7/8	0	0	3/4	59/4

	x_1	x_2	u_1	u_2	u_3	u_4	
x_1	1	0	0	0	0	1	1
x_2	0	1	1/8	0	0	-3/4	5/4
u_2	0	0	0	1	0	-1	1
u_3	0	0	1/8	0	1	-3/4	1/4
	0	0	7/8	0	0	3/4	59/4

Left child:

$z = 59/4 = 14\frac{3}{4}$
$x_1 = 1$ $x_2 = 5/4 = 1\frac{1}{4}$

	x_1	x_2	u_1	u_2	u_3	
x_1	1	0	1/6	0	4/3	4/3
x_2	0	1	0	0	-1	1
u_2	0	0	-1/6	1	-4/3	2/3
	0	0	1	0	1	15

Add constraint: $x_1 \geq 2$

	x_1	x_2	u_1	u_2	u_3	u_4	
x_1	1	0	1/6	0	4/3	0	4/3
x_2	0	1	0	0	-1	0	1
u_2	0	0	-1/6	1	-4/3	0	2/3
u_4	-1	0	0	0	0	1	-2
	0	0	1	0	1	0	15

	x_1	x_2	u_1	u_2	u_3	u_4	
x_1	1	0	1/6	0	4/3	0	4/3
x_2	0	1	0	0	-1	0	1
u_2	0	0	-1/6	1	-4/3	0	2/3
u_4	-1	0	0	0	0	1	-2
	0	0	1	0	1	0	15

	x_1	x_2	u_1	u_2	u_3	u_4	
x_1	1	0	1/6	0	4/3	0	4/3
x_2	0	1	0	0	-1	0	1
u_2	0	0	-1/6	1	-4/3	0	2/3
u_4	0	0	1/6	0	4/3	1	-2/3
	0	0	1	0	1	0	15

	x_1	x_2	u_1	u_2	u_3	u_4	
x_1	1	0	1/6	0	4/3	0	4/3
x_2	0	1	0	0	-1	0	1
u_2	0	0	-1/6	1	-4/3	0	2/3
u_4	0	0	1/6	0	4/3	1	-2/3
	0	0	1	0	1	0	15

Right child: $\emptyset$

Branch the leaf:

$z = 59/4 = 14\frac{3}{4}$
$x_1 = 1$ $x_2 = 5/4 = 1\frac{1}{4}$

New constraints: $x_2 \leq 1$ and $x_2 \geq 2$

	x_1	x_2	u_1	u_2	u_3	u_4	
x_1	1	0	0	0	0	1	1
x_2	0	1	1/8	0	0	-3/4	5/4
u_2	0	0	0	1	0	-1	1
u_3	0	0	1/8	0	1	-3/4	1/4
	0	0	7/8	0	0	3/4	59/4

Add constraint: $x_2 \leq 1$

	x_1	x_2	u_1	u_2	u_3	u_4	u_5	
x_1	1	0	0	0	0	1	0	1
x_2	0	1	1/8	0	0	-3/4	0	5/4
u_2	0	0	0	1	0	-1	0	1
u_3	0	0	1/8	0	1	-3/4	0	1/4
u_5	0	1	0	0	0	0	1	1
	0	0	7/8	0	0	3/4	0	59/4

	x_1	x_2	u_1	u_2	u_3	u_4	u_5	
x_1	1	0	0	0	0	1	0	1
x_2	0	1	1/8	0	0	-3/4	0	5/4
u_2	0	0	0	1	0	-1	0	1
u_3	0	0	1/8	0	1	-3/4	0	1/4
u_5	0	1	0	0	0	0	1	1
	0	0	7/8	0	0	3/4	0	59/4

	x_1	x_2	u_1	u_2	u_3	u_4	u_5	
x_1	1	0	0	0	0	1	0	1
x_2	0	1	1/8	0	0	-3/4	0	5/4
u_2	0	0	0	1	0	-1	0	1
u_3	0	0	1/8	0	1	-3/4	0	1/4
u_5	0	0	-1/8	0	0	3/4	1	-1/4
	0	0	7/8	0	0	3/4	0	59/4

	x_1	x_2	u_1	u_2	u_3	u_4	u_5	
x_1	1	0	0	0	0	1	0	1
x_2	0	1	1/8	0	0	-3/4	0	5/4
u_2	0	0	0	1	0	-1	0	1
u_3	0	0	1/8	0	1	-3/4	0	1/4
← u_5	0	0	-1/8	0	0	3/4	1	-1/4
	0	0	7/8	0	0	3/4	0	59/4

	x_1	x_2	u_1	u_2	u_3	u_4	u_5	
x_1	1	0	0	0	0	1	0	1
x_2	0	1	1/8	0	0	-3/4	0	5/4
u_2	0	0	0	1	0	-1	0	1
u_3	0	0	1/8	0	1	-3/4	0	1/4
← u_5	0	0	-1/8	0	0	3/4	1	-1/4
	0	0	7/8	0	0	3/4	0	59/4

	x_1	x_2	u_1	u_2	u_3	u_4	u_5	
x_1	1	0	0	0	0	1	0	1
x_2	0	1	1/8	0	0	-3/4	0	5/4
u_2	0	0	0	1	0	-1	0	1
u_3	0	0	1/8	0	1	-3/4	0	1/4
←	0	0	1	0	0	-6	-8	2
	0	0	7/8	0	0	3/4	0	59/4

	x_1	x_2	u_1	u_2	u_3	u_4	u_5	
x_1	1	0	0	0	0	1	0	1
x_2	0	1	0	0	0	0	1	1
u_2	0	0	0	1	0	-1	0	1
u_3	0	0	0	0	1	0	1	0
u_1	0	0	1	0	0	-6	-8	2
	0	0	0	0	0	6	7	13

	x_1	x_2	u_1	u_2	u_3	u_4	u_5	
x_1	1	0	0	0	0	1	0	1
x_2	0	1	0	0	0	0	1	1
u_2	0	0	0	1	0	-1	0	1
u_3	0	0	0	0	1	0	1	0
u_1	0	0	1	0	0	-6	-8	2
	0	0	0	0	0	6	7	13

Left child:

$z = 13$
$x_1 = 1$
$x_2 = 1$

	x_1	x_2	u_1	u_2	u_3	u_4	
x_1	1	0	0	0	0	1	1
x_2	0	1	1/8	0	0	-3/4	5/4
u_2	0	0	0	1	0	-1	1
u_3	0	0	1/8	0	1	-3/4	1/4
	0	0	7/8	0	0	3/4	59/4

Add constraint: $x_2 \geq 2$

	x_1	x_2	u_1	u_2	u_3	u_4	u_5	
x_1	1	0	0	0	0	1	0	1
x_2	0	1	1/8	0	0	-3/4	0	5/4
u_2	0	0	0	1	0	-1	0	1
u_3	0	0	1/8	0	1	-3/4	0	1/4
u_5	0	-1	0	0	0	0	1	-2
	0	0	7/8	0	0	3/4	0	59/4

	x_1	x_2	u_1	u_2	u_3	u_4	u_5	
x_1	1	0	0	0	0	1	0	1
x_2	0	1	1/8	0	0	-3/4	0	5/4
u_2	0	0	0	1	0	-1	0	1
u_3	0	0	1/8	0	1	-3/4	0	1/4
u_5	0	-1	0	0	0	0	1	-2
	0	0	7/8	0	0	3/4	0	59/4

	x_1	x_2	u_1	u_2	u_3	u_4	u_5	
x_1	1	0	0	0	0	1	0	1
x_2	0	1	1/8	0	0	-3/4	0	5/4
u_2	0	0	0	1	0	-1	0	1
u_3	0	0	1/8	0	1	-3/4	0	1/4
u_5	0	0	1/8	0	0	-3/4	1	-3/4
	0	0	7/8	0	0	3/4	0	59/4

	x_1	x_2	u_1	u_2	u_3	u_4	u_5	
x_1	1	0	0	0	0	1	0	1
x_2	0	1	1/8	0	0	-3/4	0	5/4
u_2	0	0	0	1	0	-1	0	1
u_3	0	0	1/8	0	1	-3/4	0	1/4
← u_5	0	0	1/8	0	0	-3/4	1	-3/4
	0	0	7/8	0	0	3/4	0	59/4

	x_1	x_2	u_1	u_2	u_3	u_4	u_5	
x_1	1	0	0	0	0	1	0	1
x_2	0	1	1/8	0	0	-3/4	0	5/4
u_2	0	0	0	1	0	-1	0	1
u_3	0	0	1/8	0	1	-3/4	0	1/4
← u_5	0	0	1/8	0	0	-3/4	1	-3/4
	0	0	7/8	0	0	3/4	0	59/4

	x_1	x_2	u_1	u_2	u_3	u_4	u_5	
x_1	1	0	0	0	0	1	0	1
x_2	0	1	1/8	0	0	-3/4	0	5/4
u_2	0	0	0	1	0	-1	0	1
u_3	0	0	1/8	0	1	-3/4	0	1/4
←	0	0	-1/6	0	0	1	-4/3	1
	0	0	7/8	0	0	3/4	0	59/4

	x_1	x_2	u_1	u_2	u_3	u_4	u_5	
x_1	1	0	1/6	0	0	0	4/3	0
x_2	0	1	0	0	0	0	-1	2
u_2	0	0	-1/6	1	0	0	-4/3	2
u_3	0	0	0	0	1	0	-1	1
u_4	0	0	-1/6	0	0	1	-4/3	1
	0	0	1	0	0	0	1	14

	x_1	x_2	u_1	u_2	u_3	u_4	u_5	
x_1	1	0	1/6	0	0	0	4/3	0
x_2	0	1	0	0	0	0	-1	2
u_2	0	0	-1/6	1	0	0	-4/3	2
u_3	0	0	0	0	1	0	-1	1
u_4	0	0	-1/6	0	0	1	-4/3	1
	0	0	1	0	0	0	1	14

Right child:

$z = 14$
$x_1 = 0$
$x_2 = 2$