

### Integer Problem:

Maximize  $z = 9x_1 + 10x_2$

subject to

$$9x_1 + 11x_2 \leq 33$$

$$(x_1, x_2) \geq 0 \text{ in } \mathbb{Z}^2$$

### Canonical Form:

Maximize  $z = 9x_1 + 10x_2$

subject to

$$9x_1 + 11x_2 + u_1 = 33$$

$$(x_1, x_2, u_1) \geq 0 \text{ in } \mathbb{Z}^3$$

	$x_1$	$x_2$	$u_1$	
$u_1$	9	11	1	33
	-9	-10	0	0

	$x_1$	$x_2$	$u_1$	
$u_1$	9	11	1	33
	-9	-10	0	0

	$x_1$	$x_2$	$u_1$	
$\leftarrow u_1$	9	11	1	33
	-9	-10	0	0

	$x_1$	$x_2$	$u_1$	
←	①	11/9	1/9	11/3
	-9	-10	0	0

	$x_1$	$x_2$	$u_1$	
$x_1$	①	11/9	1/9	11/3
	0	1	1	33

	$x_1$	$x_2$	$u_1$	
$x_1$	1	11/9	1/9	11/3
	0	1	1	33

Root of tree:

$z = 33$
$x_1 = 11/3 = 3\frac{2}{3}$
$x_2 = 0$



Branch the leaf:

$z = 33$
$x_1 = 11/3 = 3\frac{2}{3}$
$x_2 = 0$

New constraints:  $x_1 \leq 3$  and  $x_1 \geq 4$

	$x_1$	$x_2$	$u_1$	
$x_1$	1	11/9	1/9	11/3
	0	1	1	33

Add constraint:  $x_1 \leq 3$

	$x_1$	$x_2$	$u_1$	$u_2$	
$x_1$	1	11/9	1/9	0	11/3
$u_2$	1	0	0	1	3
	0	1	1	0	33

	$x_1$	$x_2$	$u_1$	$u_2$	
$x_1$	①	11/9	1/9	0	11/3
$u_2$	1	0	0	①	3
	0	1	1	0	33

	$x_1$	$x_2$	$u_1$	$u_2$	
$x_1$	①	11/9	1/9	0	11/3
$u_2$	0	-11/9	-1/9	①	-2/3
	0	1	1	0	33

	$x_1$	$x_2$	$u_1$	$u_2$	
$x_1$	1	11/9	1/9	0	11/3
$\leftarrow u_2$	0	-11/9	-1/9	1	-2/3
	0	1	1	0	33

	$x_1$	$x_2$	$u_1$	$u_2$	
$x_1$	1	11/9	1/9	0	11/3
$u_2$	0	-11/9	-1/9	1	-2/3
	0	1	1	0	33

	$x_1$	$x_2$	$u_1$	$u_2$	
$x_1$	1	11/9	1/9	0	11/3
	0	①	1/11	-9/11	6/11
	0	1	1	0	33



	$x_1$	$x_2$	$u_1$	$u_2$	
$x_1$	1	0	0	1	3
$x_2$	0	①	1/11	-9/11	6/11
	0	0	10/11	9/11	357/11

	$x_1$	$x_2$	$u_1$	$u_2$	
$x_1$	1	0	0	1	3
$x_2$	0	1	1/11	-9/11	6/11
	0	0	10/11	9/11	357/11

Left child:

$z = 357/11 = 32\frac{5}{11}$
$x_1 = 3$
$x_2 = 6/11 = \frac{6}{11}$

	$x_1$	$x_2$	$u_1$	
$x_1$	1	11/9	1/9	11/3
	0	1	1	33

Add constraint:  $x_1 \geq 4$

	$x_1$	$x_2$	$u_1$	$u_2$	
$x_1$	1	11/9	1/9	0	11/3
$u_2$	-1	0	0	1	-4
	0	1	1	0	33

	$x_1$	$x_2$	$u_1$	$u_2$	
$x_1$	①	11/9	1/9	0	11/3
$u_2$	-1	0	0	①	-4
	0	1	1	0	33

	$x_1$	$x_2$	$u_1$	$u_2$	
$x_1$	①	11/9	1/9	0	11/3
$u_2$	0	11/9	1/9	①	-1/3
	0	1	1	0	33

	$x_1$	$x_2$	$u_1$	$u_2$	
$x_1$	1	11/9	1/9	0	11/3
$u_2$	0	11/9	1/9	1	-1/3
	0	1	1	0	33

Right child:  $\emptyset$

Branch the leaf:

$z = 357/11 = 32\frac{5}{11}$
$x_1 = 3$
$x_2 = 6/11 = \frac{6}{11}$

New constraints:  $x_2 \leq 0$  and  $x_2 \geq 1$



	$x_1$	$x_2$	$u_1$	$u_2$	
$x_1$	1	0	0	1	3
$x_2$	0	1	1/11	-9/11	6/11
	0	0	10/11	9/11	357/11

Add constraint:  $x_2 \leq 0$

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	
$x_1$	1	0	0	1	0	3
$x_2$	0	1	1/11	-9/11	0	6/11
$u_3$	0	1	0	0	1	0
	0	0	10/11	9/11	0	357/11

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	
$x_1$	①	0	0	1	0	3
$x_2$	0	①	1/11	-9/11	0	6/11
$u_3$	0	1	0	0	①	0
	0	0	10/11	9/11	0	357/11

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	
$x_1$	①	0	0	1	0	3
$x_2$	0	①	1/11	-9/11	0	6/11
$u_3$	0	0	-1/11	9/11	①	-6/11
	0	0	10/11	9/11	0	357/11

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	
$x_1$	1	0	0	1	0	3
$x_2$	0	1	1/11	-9/11	0	6/11
$\leftarrow u_3$	0	0	-1/11	9/11	1	-6/11
	0	0	10/11	9/11	0	357/11

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	
$x_1$	1	0	0	1	0	3
$x_2$	0	1	1/11	-9/11	0	6/11
$u_3$	0	0	-1/11	9/11	1	-6/11
	0	0	10/11	9/11	0	357/11

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	
$x_1$	1	0	0	1	0	3
$x_2$	0	1	1/11	-9/11	0	6/11
	0	0	①	-9	-11	6
	0	0	10/11	9/11	0	357/11

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	
$x_1$	1	0	0	1	0	3
$x_2$	0	1	0	0	1	0
$u_1$	0	0	①	-9	-11	6
	0	0	0	9	10	27



	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	
$x_1$	1	0	0	1	0	3
$x_2$	0	1	0	0	1	0
$u_1$	0	0	1	-9	-11	6
	0	0	0	9	10	27

Left child:

$z = 27$
$x_1 = 3$
$x_2 = 0$

	$x_1$	$x_2$	$u_1$	$u_2$	
$x_1$	1	0	0	1	3
$x_2$	0	1	1/11	-9/11	6/11
	0	0	10/11	9/11	357/11

Add constraint:  $x_2 \geq 1$

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	
$x_1$	1	0	0	1	0	3
$x_2$	0	1	1/11	-9/11	0	6/11
$u_3$	0	-1	0	0	1	-1
	0	0	10/11	9/11	0	357/11

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	
$x_1$	①	0	0	1	0	3
$x_2$	0	①	1/11	-9/11	0	6/11
$u_3$	0	-1	0	0	①	-1
	0	0	10/11	9/11	0	357/11

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	
$x_1$	①	0	0	1	0	3
$x_2$	0	①	1/11	-9/11	0	6/11
$u_3$	0	0	1/11	-9/11	①	-5/11
	0	0	10/11	9/11	0	357/11

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	
$x_1$	1	0	0	1	0	3
$x_2$	0	1	1/11	-9/11	0	6/11
$\leftarrow u_3$	0	0	1/11	-9/11	1	-5/11
	0	0	10/11	9/11	0	357/11

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	
$x_1$	1	0	0	1	0	3
$x_2$	0	1	1/11	-9/11	0	6/11
$u_3$	0	0	1/11	-9/11	1	-5/11
	0	0	10/11	9/11	0	357/11

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	
$x_1$	1	0	0	1	0	3
$x_2$	0	1	1/11	-9/11	0	6/11
	0	0	-1/9	①	-11/9	5/9
	0	0	10/11	9/11	0	357/11



	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	
$x_1$	1	0	1/9	0	11/9	22/9
$x_2$	0	1	0	0	-1	1
$u_2$	0	0	-1/9	1	-11/9	5/9
	0	0	1	0	1	32

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	
$x_1$	1	0	1/9	0	11/9	22/9
$x_2$	0	1	0	0	-1	1
$u_2$	0	0	-1/9	1	-11/9	5/9
	0	0	1	0	1	32

Right child:

$z = 32$
$x_1 = 22/9 = 2\frac{4}{9}$
$x_2 = 1$

Branch the leaf:

$z = 32$
$x_1 = 22/9 = 2\frac{4}{9}$
$x_2 = 1$

New constraints:  $x_1 \leq 2$  and  $x_1 \geq 3$

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	
$x_1$	1	0	1/9	0	11/9	22/9
$x_2$	0	1	0	0	-1	1
$u_2$	0	0	-1/9	1	-11/9	5/9
	0	0	1	0	1	32

Add constraint:  $x_1 \leq 2$

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	
$x_1$	1	0	1/9	0	11/9	0	22/9
$x_2$	0	1	0	0	-1	0	1
$u_2$	0	0	-1/9	1	-11/9	0	5/9
$u_4$	1	0	0	0	0	1	2
	0	0	1	0	1	0	32

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	
$x_1$	①	0	1/9	0	11/9	0	22/9
$x_2$	0	①	0	0	-1	0	1
$u_2$	0	0	-1/9	①	-11/9	0	5/9
$u_4$	1	0	0	0	0	①	2
	0	0	1	0	1	0	32

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	
$x_1$	①	0	1/9	0	11/9	0	22/9
$x_2$	0	①	0	0	-1	0	1
$u_2$	0	0	-1/9	①	-11/9	0	5/9
$u_4$	0	0	-1/9	0	-11/9	①	-4/9
	0	0	1	0	1	0	32

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	
$x_1$	1	0	1/9	0	11/9	0	22/9
$x_2$	0	1	0	0	-1	0	1
$u_2$	0	0	-1/9	1	-11/9	0	5/9
$\leftarrow u_4$	0	0	-1/9	0	-11/9	1	-4/9
	0	0	1	0	1	0	32



	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	
$x_1$	1	0	1/9	0	11/9	0	22/9
$x_2$	0	1	0	0	-1	0	1
$u_2$	0	0	-1/9	1	-11/9	0	5/9
$u_4$	0	0	-1/9	0	-11/9	1	-4/9
	0	0	1	0	1	0	32

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	
$x_1$	1	0	1/9	0	11/9	0	22/9
$x_2$	0	1	0	0	-1	0	1
$u_2$	0	0	-1/9	1	-11/9	0	5/9
←	0	0	1/11	0	①	-9/11	4/11
	0	0	1	0	1	0	32

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	
$x_1$	1	0	0	0	0	1	2
$x_2$	0	1	1/11	0	0	-9/11	15/11
$u_2$	0	0	0	1	0	-1	1
$u_3$	0	0	1/11	0	①	-9/11	4/11
	0	0	10/11	0	0	9/11	348/11

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	
$x_1$	1	0	0	0	0	1	2
$x_2$	0	1	1/11	0	0	-9/11	15/11
$u_2$	0	0	0	1	0	-1	1
$u_3$	0	0	1/11	0	1	-9/11	4/11
	0	0	10/11	0	0	9/11	348/11

Left child:

$z = 348/11 = 31\frac{7}{11}$
$x_1 = 2$
$x_2 = 15/11 = 1\frac{4}{11}$

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	
$x_1$	1	0	1/9	0	11/9	22/9
$x_2$	0	1	0	0	-1	1
$u_2$	0	0	-1/9	1	-11/9	5/9
	0	0	1	0	1	32

Add constraint:  $x_1 \geq 3$

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	
$x_1$	1	0	1/9	0	11/9	0	22/9
$x_2$	0	1	0	0	-1	0	1
$u_2$	0	0	-1/9	1	-11/9	0	5/9
$u_4$	-1	0	0	0	0	1	-3
	0	0	1	0	1	0	32

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	
$x_1$	①	0	1/9	0	11/9	0	22/9
$x_2$	0	①	0	0	-1	0	1
$u_2$	0	0	-1/9	①	-11/9	0	5/9
$u_4$	-1	0	0	0	0	①	-3
	0	0	1	0	1	0	32

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	
$x_1$	①	0	1/9	0	11/9	0	22/9
$x_2$	0	①	0	0	-1	0	1
$u_2$	0	0	-1/9	①	-11/9	0	5/9
$u_4$	0	0	1/9	0	11/9	①	-5/9
	0	0	1	0	1	0	32



	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	
$x_1$	1	0	1/9	0	11/9	0	22/9
$x_2$	0	1	0	0	-1	0	1
$u_2$	0	0	-1/9	1	-11/9	0	5/9
$u_4$	0	0	1/9	0	11/9	1	-5/9
	0	0	1	0	1	0	32

Right child:  $\emptyset$

Branch the leaf:

$z = 348/11 = 31\frac{7}{11}$
$x_1 = 2$
$x_2 = 15/11 = 1\frac{4}{11}$

New constraints:  $x_2 \leq 1$  and  $x_2 \geq 2$

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	
$x_1$	1	0	0	0	0	1	2
$x_2$	0	1	1/11	0	0	-9/11	15/11
$u_2$	0	0	0	1	0	-1	1
$u_3$	0	0	1/11	0	1	-9/11	4/11
	0	0	10/11	0	0	9/11	348/11

Add constraint:  $x_2 \leq 1$

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	
$x_1$	1	0	0	0	0	1	0	2
$x_2$	0	1	1/11	0	0	-9/11	0	15/11
$u_2$	0	0	0	1	0	-1	0	1
$u_3$	0	0	1/11	0	1	-9/11	0	4/11
$u_5$	0	1	0	0	0	0	1	1
	0	0	10/11	0	0	9/11	0	348/11

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	
$x_1$	①	0	0	0	0	1	0	2
$x_2$	0	①	1/11	0	0	-9/11	0	15/11
$u_2$	0	0	0	①	0	-1	0	1
$u_3$	0	0	1/11	0	①	-9/11	0	4/11
$u_5$	0	1	0	0	0	0	①	1
	0	0	10/11	0	0	9/11	0	348/11

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	
$x_1$	①	0	0	0	0	1	0	2
$x_2$	0	①	1/11	0	0	-9/11	0	15/11
$u_2$	0	0	0	①	0	-1	0	1
$u_3$	0	0	1/11	0	①	-9/11	0	4/11
$u_5$	0	0	-1/11	0	0	9/11	①	-4/11
	0	0	10/11	0	0	9/11	0	348/11

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	
$x_1$	1	0	0	0	0	1	0	2
$x_2$	0	1	1/11	0	0	-9/11	0	15/11
$u_2$	0	0	0	1	0	-1	0	1
$u_3$	0	0	1/11	0	1	-9/11	0	4/11
$\leftarrow u_5$	0	0	-1/11	0	0	9/11	1	-4/11
	0	0	10/11	0	0	9/11	0	348/11

	$x_1$	$x_2$	$\downarrow$ $u_1$	$u_2$	$u_3$	$u_4$	$u_5$	
$x_1$	1	0	0	0	0	1	0	2
$x_2$	0	1	1/11	0	0	-9/11	0	15/11
$u_2$	0	0	0	1	0	-1	0	1
$u_3$	0	0	1/11	0	1	-9/11	0	4/11
$\leftarrow u_5$	0	0	-1/11	0	0	9/11	1	-4/11
	0	0	10/11	0	0	9/11	0	348/11



	$x_1$	$x_2$	$\downarrow$ $u_1$	$u_2$	$u_3$	$u_4$	$u_5$	
$x_1$	1	0	0	0	0	1	0	2
$x_2$	0	1	1/11	0	0	-9/11	0	15/11
$u_2$	0	0	0	1	0	-1	0	1
$u_3$	0	0	1/11	0	1	-9/11	0	4/11
$\leftarrow$	0	0	①	0	0	-9	-11	4
	0	0	10/11	0	0	9/11	0	348/11

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	
$x_1$	1	0	0	0	0	1	0	2
$x_2$	0	1	0	0	0	0	1	1
$u_2$	0	0	0	1	0	-1	0	1
$u_3$	0	0	0	0	1	0	1	0
$u_1$	0	0	①	0	0	-9	-11	4
	0	0	0	0	0	9	10	28

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	
$x_1$	1	0	0	0	0	1	0	2
$x_2$	0	1	0	0	0	0	1	1
$u_2$	0	0	0	1	0	-1	0	1
$u_3$	0	0	0	0	1	0	1	0
$u_1$	0	0	1	0	0	-9	-11	4
	0	0	0	0	0	9	10	28

Left child:

$z = 28$
$x_1 = 2$
$x_2 = 1$

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	
$x_1$	1	0	0	0	0	1	2
$x_2$	0	1	1/11	0	0	-9/11	15/11
$u_2$	0	0	0	1	0	-1	1
$u_3$	0	0	1/11	0	1	-9/11	4/11
	0	0	10/11	0	0	9/11	348/11

Add constraint:  $x_2 \geq 2$

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	
$x_1$	1	0	0	0	0	1	0	2
$x_2$	0	1	1/11	0	0	-9/11	0	15/11
$u_2$	0	0	0	1	0	-1	0	1
$u_3$	0	0	1/11	0	1	-9/11	0	4/11
$u_5$	0	-1	0	0	0	0	1	-2
	0	0	10/11	0	0	9/11	0	348/11

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	
$x_1$	①	0	0	0	0	1	0	2
$x_2$	0	①	1/11	0	0	-9/11	0	15/11
$u_2$	0	0	0	①	0	-1	0	1
$u_3$	0	0	1/11	0	①	-9/11	0	4/11
$u_5$	0	-1	0	0	0	0	①	-2
	0	0	10/11	0	0	9/11	0	348/11

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	
$x_1$	①	0	0	0	0	1	0	2
$x_2$	0	①	1/11	0	0	-9/11	0	15/11
$u_2$	0	0	0	①	0	-1	0	1
$u_3$	0	0	1/11	0	①	-9/11	0	4/11
$u_5$	0	0	1/11	0	0	-9/11	①	-7/11
	0	0	10/11	0	0	9/11	0	348/11

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	
$x_1$	1	0	0	0	0	1	0	2
$x_2$	0	1	1/11	0	0	-9/11	0	15/11
$u_2$	0	0	0	1	0	-1	0	1
$u_3$	0	0	1/11	0	1	-9/11	0	4/11
$\leftarrow u_5$	0	0	1/11	0	0	-9/11	1	-7/11
	0	0	10/11	0	0	9/11	0	348/11



	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	
$x_1$	1	0	0	0	0	1	0	2
$x_2$	0	1	1/11	0	0	-9/11	0	15/11
$u_2$	0	0	0	1	0	-1	0	1
$u_3$	0	0	1/11	0	1	-9/11	0	4/11
$\leftarrow u_5$	0	0	1/11	0	0	-9/11	1	-7/11
	0	0	10/11	0	0	9/11	0	348/11

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	
$x_1$	1	0	0	0	0	1	0	2
$x_2$	0	1	1/11	0	0	-9/11	0	15/11
$u_2$	0	0	0	1	0	-1	0	1
$u_3$	0	0	1/11	0	1	-9/11	0	4/11
←	0	0	-1/9	0	0	①	-11/9	7/9
	0	0	10/11	0	0	9/11	0	348/11

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	
$x_1$	1	0	1/9	0	0	0	11/9	11/9
$x_2$	0	1	0	0	0	0	-1	2
$u_2$	0	0	-1/9	1	0	0	-11/9	16/9
$u_3$	0	0	0	0	1	0	-1	1
$u_4$	0	0	-1/9	0	0	①	-11/9	7/9
	0	0	1	0	0	0	1	31

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	
$x_1$	1	0	1/9	0	0	0	11/9	11/9
$x_2$	0	1	0	0	0	0	-1	2
$u_2$	0	0	-1/9	1	0	0	-11/9	16/9
$u_3$	0	0	0	0	1	0	-1	1
$u_4$	0	0	-1/9	0	0	1	-11/9	7/9
	0	0	1	0	0	0	1	31

Right child:

$z = 31$
$x_1 = 11/9 = 1\frac{2}{9}$
$x_2 = 2$

Branch the leaf:

$z = 31$
$x_1 = 11/9 = 1\frac{2}{9}$
$x_2 = 2$

New constraints:  $x_1 \leq 1$  and  $x_1 \geq 2$

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	
$x_1$	1	0	1/9	0	0	0	11/9	11/9
$x_2$	0	1	0	0	0	0	-1	2
$u_2$	0	0	-1/9	1	0	0	-11/9	16/9
$u_3$	0	0	0	0	1	0	-1	1
$u_4$	0	0	-1/9	0	0	1	-11/9	7/9
	0	0	1	0	0	0	1	31

Add constraint:  $x_1 \leq 1$

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	
$x_1$	1	0	1/9	0	0	0	11/9	0	11/9
$x_2$	0	1	0	0	0	0	-1	0	2
$u_2$	0	0	-1/9	1	0	0	-11/9	0	16/9
$u_3$	0	0	0	0	1	0	-1	0	1
$u_4$	0	0	-1/9	0	0	1	-11/9	0	7/9
$u_6$	1	0	0	0	0	0	0	1	1
	0	0	1	0	0	0	1	0	31

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	
$x_1$	①	0	1/9	0	0	0	11/9	0	11/9
$x_2$	0	①	0	0	0	0	-1	0	2
$u_2$	0	0	-1/9	①	0	0	-11/9	0	16/9
$u_3$	0	0	0	0	①	0	-1	0	1
$u_4$	0	0	-1/9	0	0	①	-11/9	0	7/9
$u_6$	1	0	0	0	0	0	0	①	1
	0	0	1	0	0	0	1	0	31



	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	
$x_1$	①	0	1/9	0	0	0	11/9	0	11/9
$x_2$	0	①	0	0	0	0	-1	0	2
$u_2$	0	0	-1/9	①	0	0	-11/9	0	16/9
$u_3$	0	0	0	0	①	0	-1	0	1
$u_4$	0	0	-1/9	0	0	①	-11/9	0	7/9
$u_6$	0	0	-1/9	0	0	0	-11/9	①	-2/9
	0	0	1	0	0	0	1	0	31

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	
$x_1$	1	0	1/9	0	0	0	11/9	0	11/9
$x_2$	0	1	0	0	0	0	-1	0	2
$u_2$	0	0	-1/9	1	0	0	-11/9	0	16/9
$u_3$	0	0	0	0	1	0	-1	0	1
$u_4$	0	0	-1/9	0	0	1	-11/9	0	7/9
$\leftarrow u_6$	0	0	-1/9	0	0	0	-11/9	1	-2/9
	0	0	1	0	0	0	1	0	31

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	
$x_1$	1	0	1/9	0	0	0	11/9	0	11/9
$x_2$	0	1	0	0	0	0	-1	0	2
$u_2$	0	0	-1/9	1	0	0	-11/9	0	16/9
$u_3$	0	0	0	0	1	0	-1	0	1
$u_4$	0	0	-1/9	0	0	1	-11/9	0	7/9
$u_6$	0	0	-1/9	0	0	0	-11/9	1	-2/9
	0	0	1	0	0	0	1	0	31

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	
$x_1$	1	0	1/9	0	0	0	11/9	0	11/9
$x_2$	0	1	0	0	0	0	-1	0	2
$u_2$	0	0	-1/9	1	0	0	-11/9	0	16/9
$u_3$	0	0	0	0	1	0	-1	0	1
$u_4$	0	0	-1/9	0	0	1	-11/9	0	7/9
$u_5$	0	0	1/11	0	0	0	1	-9/11	2/11
	0	0	1	0	0	0	1	0	31

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	
$x_1$	1	0	0	0	0	0	0	1	1
$x_2$	0	1	1/11	0	0	0	0	-9/11	24/11
$u_2$	0	0	0	1	0	0	0	-1	2
$u_3$	0	0	1/11	0	1	0	0	-9/11	13/11
$u_4$	0	0	0	0	0	1	0	-1	1
$u_5$	0	0	1/11	0	0	0	①	-9/11	2/11
	0	0	10/11	0	0	0	0	9/11	339/11

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	
$x_1$	1	0	0	0	0	0	0	1	1
$x_2$	0	1	1/11	0	0	0	0	-9/11	24/11
$u_2$	0	0	0	1	0	0	0	-1	2
$u_3$	0	0	1/11	0	1	0	0	-9/11	13/11
$u_4$	0	0	0	0	0	1	0	-1	1
$u_5$	0	0	1/11	0	0	0	1	-9/11	2/11
	0	0	10/11	0	0	0	0	9/11	339/11

Left child:

$z = 339/11 = 30\frac{9}{11}$
$x_1 = 1$
$x_2 = 24/11 = 2\frac{2}{11}$

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	
$x_1$	1	0	1/9	0	0	0	11/9	11/9
$x_2$	0	1	0	0	0	0	-1	2
$u_2$	0	0	-1/9	1	0	0	-11/9	16/9
$u_3$	0	0	0	0	1	0	-1	1
$u_4$	0	0	-1/9	0	0	1	-11/9	7/9
	0	0	1	0	0	0	1	31

Add constraint:  $x_1 \geq 2$

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	
$x_1$	1	0	1/9	0	0	0	11/9	0	11/9
$x_2$	0	1	0	0	0	0	-1	0	2
$u_2$	0	0	-1/9	1	0	0	-11/9	0	16/9
$u_3$	0	0	0	0	1	0	-1	0	1
$u_4$	0	0	-1/9	0	0	1	-11/9	0	7/9
$u_6$	-1	0	0	0	0	0	0	1	-2
	0	0	1	0	0	0	1	0	31



	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	
$x_1$	①	0	1/9	0	0	0	11/9	0	11/9
$x_2$	0	①	0	0	0	0	-1	0	2
$u_2$	0	0	-1/9	①	0	0	-11/9	0	16/9
$u_3$	0	0	0	0	①	0	-1	0	1
$u_4$	0	0	-1/9	0	0	①	-11/9	0	7/9
$u_6$	-1	0	0	0	0	0	0	①	-2
	0	0	1	0	0	0	1	0	31

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	
$x_1$	①	0	1/9	0	0	0	11/9	0	11/9
$x_2$	0	①	0	0	0	0	-1	0	2
$u_2$	0	0	-1/9	①	0	0	-11/9	0	16/9
$u_3$	0	0	0	0	①	0	-1	0	1
$u_4$	0	0	-1/9	0	0	①	-11/9	0	7/9
$u_6$	0	0	1/9	0	0	0	11/9	①	-7/9
	0	0	1	0	0	0	1	0	31

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	
$x_1$	1	0	1/9	0	0	0	11/9	0	11/9
$x_2$	0	1	0	0	0	0	-1	0	2
$u_2$	0	0	-1/9	1	0	0	-11/9	0	16/9
$u_3$	0	0	0	0	1	0	-1	0	1
$u_4$	0	0	-1/9	0	0	1	-11/9	0	7/9
$u_6$	0	0	1/9	0	0	0	11/9	1	-7/9
	0	0	1	0	0	0	1	0	31

Right child:  $\emptyset$

Branch the leaf:

$z = 339/11 = 30\frac{9}{11}$
$x_1 = 1$
$x_2 = 24/11 = 2\frac{2}{11}$

New constraints:  $x_2 \leq 2$  and  $x_2 \geq 3$

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	
$x_1$	1	0	0	0	0	0	0	1	1
$x_2$	0	1	1/11	0	0	0	0	-9/11	24/11
$u_2$	0	0	0	1	0	0	0	-1	2
$u_3$	0	0	1/11	0	1	0	0	-9/11	13/11
$u_4$	0	0	0	0	0	1	0	-1	1
$u_5$	0	0	1/11	0	0	0	1	-9/11	2/11
	0	0	10/11	0	0	0	0	9/11	339/11

Add constraint:  $x_2 \leq 2$

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	$u_7$	
$x_1$	1	0	0	0	0	0	0	1	0	1
$x_2$	0	1	1/11	0	0	0	0	-9/11	0	24/11
$u_2$	0	0	0	1	0	0	0	-1	0	2
$u_3$	0	0	1/11	0	1	0	0	-9/11	0	13/11
$u_4$	0	0	0	0	0	1	0	-1	0	1
$u_5$	0	0	1/11	0	0	0	1	-9/11	0	2/11
$u_7$	0	1	0	0	0	0	0	0	1	2
	0	0	10/11	0	0	0	0	9/11	0	339/11

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	$u_7$	
$x_1$	①	0	0	0	0	0	0	1	0	1
$x_2$	0	①	1/11	0	0	0	0	-9/11	0	24/11
$u_2$	0	0	0	①	0	0	0	-1	0	2
$u_3$	0	0	1/11	0	①	0	0	-9/11	0	13/11
$u_4$	0	0	0	0	0	①	0	-1	0	1
$u_5$	0	0	1/11	0	0	0	①	-9/11	0	2/11
$u_7$	0	1	0	0	0	0	0	0	①	2
	0	0	10/11	0	0	0	0	9/11	0	339/11

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	$u_7$	
$x_1$	①	0	0	0	0	0	0	1	0	1
$x_2$	0	①	1/11	0	0	0	0	-9/11	0	24/11
$u_2$	0	0	0	①	0	0	0	-1	0	2
$u_3$	0	0	1/11	0	①	0	0	-9/11	0	13/11
$u_4$	0	0	0	0	0	①	0	-1	0	1
$u_5$	0	0	1/11	0	0	0	①	-9/11	0	2/11
$u_7$	0	0	-1/11	0	0	0	0	9/11	①	-2/11
	0	0	10/11	0	0	0	0	9/11	0	339/11



	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	$u_7$	
$x_1$	1	0	0	0	0	0	0	1	0	1
$x_2$	0	1	1/11	0	0	0	0	-9/11	0	24/11
$u_2$	0	0	0	1	0	0	0	-1	0	2
$u_3$	0	0	1/11	0	1	0	0	-9/11	0	13/11
$u_4$	0	0	0	0	0	1	0	-1	0	1
$u_5$	0	0	1/11	0	0	0	1	-9/11	0	2/11
$\leftarrow u_7$	0	0	-1/11	0	0	0	0	9/11	1	-2/11
	0	0	10/11	0	0	0	0	9/11	0	339/11

	$x_1$	$x_2$	$\downarrow$ $u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	$u_7$	
$x_1$	1	0	0	0	0	0	0	1	0	1
$x_2$	0	1	1/11	0	0	0	0	-9/11	0	24/11
$u_2$	0	0	0	1	0	0	0	-1	0	2
$u_3$	0	0	1/11	0	1	0	0	-9/11	0	13/11
$u_4$	0	0	0	0	0	1	0	-1	0	1
$u_5$	0	0	1/11	0	0	0	1	-9/11	0	2/11
$\leftarrow u_7$	0	0	-1/11	0	0	0	0	9/11	1	-2/11
	0	0	10/11	0	0	0	0	9/11	0	339/11

	$x_1$	$x_2$	$\downarrow$ $u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	$u_7$	
$x_1$	1	0	0	0	0	0	0	1	0	1
$x_2$	0	1	1/11	0	0	0	0	-9/11	0	24/11
$u_2$	0	0	0	1	0	0	0	-1	0	2
$u_3$	0	0	1/11	0	1	0	0	-9/11	0	13/11
$u_4$	0	0	0	0	0	1	0	-1	0	1
$u_5$	0	0	1/11	0	0	0	1	-9/11	0	2/11
$\leftarrow$	0	0	①	0	0	0	0	-9	-11	2
	0	0	10/11	0	0	0	0	9/11	0	339/11

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	$u_7$	
$x_1$	1	0	0	0	0	0	0	1	0	1
$x_2$	0	1	0	0	0	0	0	0	1	2
$u_2$	0	0	0	1	0	0	0	-1	0	2
$u_3$	0	0	0	0	1	0	0	0	1	1
$u_4$	0	0	0	0	0	1	0	-1	0	1
$u_5$	0	0	0	0	0	0	1	0	1	0
$u_1$	0	0	①	0	0	0	0	-9	-11	2
	0	0	0	0	0	0	0	9	10	29

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	$u_7$	
$x_1$	1	0	0	0	0	0	0	1	0	1
$x_2$	0	1	0	0	0	0	0	0	1	2
$u_2$	0	0	0	1	0	0	0	-1	0	2
$u_3$	0	0	0	0	1	0	0	0	1	1
$u_4$	0	0	0	0	0	1	0	-1	0	1
$u_5$	0	0	0	0	0	0	1	0	1	0
$u_1$	0	0	1	0	0	0	0	-9	-11	2
	0	0	0	0	0	0	0	9	10	29

Left child:

$z = 29$
$x_1 = 1$
$x_2 = 2$

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	
$x_1$	1	0	0	0	0	0	0	1	1
$x_2$	0	1	1/11	0	0	0	0	-9/11	24/11
$u_2$	0	0	0	1	0	0	0	-1	2
$u_3$	0	0	1/11	0	1	0	0	-9/11	13/11
$u_4$	0	0	0	0	0	1	0	-1	1
$u_5$	0	0	1/11	0	0	0	1	-9/11	2/11
	0	0	10/11	0	0	0	0	9/11	339/11

Add constraint:  $x_2 \geq 3$

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	$u_7$	
$x_1$	1	0	0	0	0	0	0	1	0	1
$x_2$	0	1	1/11	0	0	0	0	-9/11	0	24/11
$u_2$	0	0	0	1	0	0	0	-1	0	2
$u_3$	0	0	1/11	0	1	0	0	-9/11	0	13/11
$u_4$	0	0	0	0	0	1	0	-1	0	1
$u_5$	0	0	1/11	0	0	0	1	-9/11	0	2/11
$u_7$	0	-1	0	0	0	0	0	0	1	-3
	0	0	10/11	0	0	0	0	9/11	0	339/11

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	$u_7$	
$x_1$	①	0	0	0	0	0	0	1	0	1
$x_2$	0	①	1/11	0	0	0	0	-9/11	0	24/11
$u_2$	0	0	0	①	0	0	0	-1	0	2
$u_3$	0	0	1/11	0	①	0	0	-9/11	0	13/11
$u_4$	0	0	0	0	0	①	0	-1	0	1
$u_5$	0	0	1/11	0	0	0	①	-9/11	0	2/11
$u_7$	0	-1	0	0	0	0	0	0	①	-3
	0	0	10/11	0	0	0	0	9/11	0	339/11



	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	$u_7$	
$x_1$	①	0	0	0	0	0	0	1	0	1
$x_2$	0	①	1/11	0	0	0	0	-9/11	0	24/11
$u_2$	0	0	0	①	0	0	0	-1	0	2
$u_3$	0	0	1/11	0	①	0	0	-9/11	0	13/11
$u_4$	0	0	0	0	0	①	0	-1	0	1
$u_5$	0	0	1/11	0	0	0	①	-9/11	0	2/11
$u_7$	0	0	1/11	0	0	0	0	-9/11	①	-9/11
	0	0	10/11	0	0	0	0	9/11	0	339/11

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	$u_7$	
$x_1$	1	0	0	0	0	0	0	1	0	1
$x_2$	0	1	1/11	0	0	0	0	-9/11	0	24/11
$u_2$	0	0	0	1	0	0	0	-1	0	2
$u_3$	0	0	1/11	0	1	0	0	-9/11	0	13/11
$u_4$	0	0	0	0	0	1	0	-1	0	1
$u_5$	0	0	1/11	0	0	0	1	-9/11	0	2/11
$\leftarrow u_7$	0	0	1/11	0	0	0	0	-9/11	1	-9/11
	0	0	10/11	0	0	0	0	9/11	0	339/11

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	$u_7$	
$x_1$	1	0	0	0	0	0	0	1	0	1
$x_2$	0	1	1/11	0	0	0	0	-9/11	0	24/11
$u_2$	0	0	0	1	0	0	0	-1	0	2
$u_3$	0	0	1/11	0	1	0	0	-9/11	0	13/11
$u_4$	0	0	0	0	0	1	0	-1	0	1
$u_5$	0	0	1/11	0	0	0	1	-9/11	0	2/11
$u_7$	0	0	1/11	0	0	0	0	-9/11	1	-9/11
	0	0	10/11	0	0	0	0	9/11	0	339/11

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	$u_7$	
$x_1$	1	0	0	0	0	0	0	1	0	1
$x_2$	0	1	1/11	0	0	0	0	-9/11	0	24/11
$u_2$	0	0	0	1	0	0	0	-1	0	2
$u_3$	0	0	1/11	0	1	0	0	-9/11	0	13/11
$u_4$	0	0	0	0	0	1	0	-1	0	1
$u_5$	0	0	1/11	0	0	0	1	-9/11	0	2/11
	0	0	-1/9	0	0	0	0	①	-11/9	1
	0	0	10/11	0	0	0	0	9/11	0	339/11

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	$u_7$	
$x_1$	1	0	1/9	0	0	0	0	0	11/9	0
$x_2$	0	1	0	0	0	0	0	0	-1	3
$u_2$	0	0	-1/9	1	0	0	0	0	-11/9	3
$u_3$	0	0	0	0	1	0	0	0	-1	2
$u_4$	0	0	-1/9	0	0	1	0	0	-11/9	2
$u_5$	0	0	0	0	0	0	1	0	-1	1
$u_6$	0	0	-1/9	0	0	0	0	1	-11/9	1
	0	0	1	0	0	0	0	0	1	30

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	$u_7$	
$x_1$	1	0	1/9	0	0	0	0	0	11/9	0
$x_2$	0	1	0	0	0	0	0	0	-1	3
$u_2$	0	0	-1/9	1	0	0	0	0	-11/9	3
$u_3$	0	0	0	0	1	0	0	0	-1	2
$u_4$	0	0	-1/9	0	0	1	0	0	-11/9	2
$u_5$	0	0	0	0	0	0	1	0	-1	1
$u_6$	0	0	-1/9	0	0	0	0	1	-11/9	1
	0	0	1	0	0	0	0	0	1	30

Right child:

$z = 30$
$x_1 = 0$
$x_2 = 3$