

### Integer Problem:

Maximize  $z = 6x_1 + 7x_2$

subject to

$$6x_1 + 8x_2 \leq 16$$

$$(x_1, x_2) \geq 0 \text{ in } \mathbb{Z}^2$$

### Canonical Form:

Maximize  $z = 6x_1 + 7x_2$

subject to

$$6x_1 + 8x_2 + u_1 = 16$$

$$(x_1, x_2, u_1) \geq 0 \text{ in } \mathbb{Z}^3$$

	$x_1$	$x_2$	$u_1$	
$u_1$	6	8	1	16
	-6	-7	0	0

 $x_1$  $x_2$  $u_1$  $u_1$ 

6	8	1	16
-6	-7	0	0

 $x_1$  $x_2$  $u_1$  $\leftarrow u_1$ 

6	8	1	16
-6	-7	0	0

 $x_1$  $x_2$  $u_1$ 

$x_1$	$x_2$	$u_1$	
1	$4/3$	$1/6$	$8/3$
-6	-7	0	0

	$x_1$	$x_2$	$u_1$	
$x_1$	1	$4/3$	$1/6$	$8/3$
	0	1	1	16

	$x_1$	$x_2$	$u_1$	
$x_1$	1	$4/3$	$1/6$	$8/3$
	0	1	1	16

$$x_1 + \frac{4}{3}x_2 + \frac{1}{6}u_1 = \frac{8}{3} \quad ; \quad (x, u) \geq 0 \text{ in } \mathbb{Z}^3$$

$$\text{Mixed integer cutting plane: } \frac{1}{3}x_2 + \frac{1}{6}u_1 \geq \frac{2}{3}$$



	$x_1$	$x_2$	$u_1$	$u_2$	
$x_1$	1	$4/3$	$1/6$	0	$8/3$
$u_2$	0	$-1/3$	$-1/6$	1	$-2/3$
	0	1	1	0	16

	$x_1$	$x_2$	$u_1$	$u_2$	
$x_1$	1	$4/3$	$1/6$	0	$8/3$
$\leftarrow u_2$	0	$-1/3$	$-1/6$	1	$-2/3$
	0	1	1	0	16



$x_1$        $x_2$        $u_1$        $u_2$

$x_1$

1	4/3	1/6	0	8/3
0	-1/3	-1/6	1	-2/3
0	1	1	0	16

$\leftarrow u_2$



$x_1$       $x_2$       $u_1$       $u_2$

$x_1$



1	$4/3$	$1/6$	0	$8/3$
0	1	$1/2$	-3	2
0	1	1	0	16

	$x_1$	$x_2$	$u_1$	$u_2$	
$x_1$	1	0	-1/2	4	0
$x_2$	0	1	1/2	-3	2
	0	0	1/2	3	14

	$x_1$	$x_2$	$u_1$	$u_2$	
$x_1$	1	0	-1/2	4	0
$x_2$	0	1	1/2	-3	2
	0	0	1/2	3	14

Optimal solution:  $(x_1, x_2) = (0, 2)$