

### Integer Problem:

Maximize  $z = 9x_1 + 10x_2$

subject to

$$9x_1 + 11x_2 \leq 33$$

$$(x_1, x_2) \geq 0 \text{ in } \mathbb{Z}^2$$

### Canonical Form:

Maximize  $z = 9x_1 + 10x_2$

subject to

$$9x_1 + 11x_2 + u_1 = 33$$

$$(x_1, x_2, u_1) \geq 0 \text{ in } \mathbb{Z}^3$$

	$x_1$	$x_2$	$u_1$	
$u_1$	9	11	1	33
	-9	-10	0	0

 $x_1$  $x_2$  $u_1$  $u_1$ 

9	11	1	33
-9	-10	0	0

 $x_1$  $x_2$  $u_1$  $\leftarrow u_1$ 

9	11	1	33
-9	-10	0	0

 $x_1$  $x_2$  $u_1$ 

	$x_1$	$x_2$	$u_1$	
	1	11/9	1/9	11/3
	-9	-10	0	0

	$x_1$	$x_2$	$u_1$	
$x_1$	1	11/9	1/9	11/3
	0	1	1	33

	$x_1$	$x_2$	$u_1$	
$x_1$	1	11/9	1/9	11/3
	0	1	1	33

$$x_1 + \frac{11}{9}x_2 + \frac{1}{9}u_1 = \frac{11}{3} \quad ; \quad (x, u) \geq 0 \text{ in } \mathbb{Z}^3$$

$$\text{Mixed integer cutting plane: } \frac{2}{9}x_2 + \frac{1}{9}u_1 \geq \frac{2}{3}$$



	$x_1$	$x_2$	$u_1$	$u_2$	
$x_1$	1	11/9	1/9	0	11/3
$u_2$	0	-2/9	-1/9	1	-2/3
	0	1	1	0	33

	$x_1$	$x_2$	$u_1$	$u_2$	
$x_1$	1	11/9	1/9	0	11/3
$\leftarrow u_2$	0	-2/9	-1/9	1	-2/3
	0	1	1	0	33



$x_1$       $x_2$       $u_1$       $u_2$

$x_1$

1	11/9	1/9	0	11/3
0	-2/9	-1/9	1	-2/3
0	1	1	0	33

$\leftarrow u_2$



$x_1$        $x_2$        $u_1$        $u_2$

$x_1$



1	11/9	1/9	0	11/3
0	1	1/2	-9/2	3
0	1	1	0	33

	$x_1$	$x_2$	$u_1$	$u_2$	
$x_1$	1	0	$-1/2$	$11/2$	0
$x_2$	0	1	$1/2$	$-9/2$	3
	0	0	$1/2$	$9/2$	30

	$x_1$	$x_2$	$u_1$	$u_2$	
$x_1$	1	0	-1/2	11/2	0
$x_2$	0	1	1/2	-9/2	3
	0	0	1/2	9/2	30

Optimal solution:  $(x_1, x_2) = (0, 3)$