

### Integer Problem:

Maximize  $z = 3x_1 + 2x_2$

subject to

$$2x_1 + x_2 \leq 12$$

$$x_1 + 3x_2 \leq 15$$

$$(x_1, x_2) \geq 0 \text{ in } \mathbb{Z}^2$$

### Canonical Form:

Maximize  $z = 3x_1 + 2x_2$

subject to

$$2x_1 + x_2 + u_1 = 12$$

$$x_1 + 3x_2 + u_2 = 15$$

$$(x_1, x_2, u_1, u_2) \geq 0 \text{ in } \mathbb{Z}^4$$

	$x_1$	$x_2$	$u_1$	$u_2$	
$u_1$	2	1	1	0	12
$u_2$	1	3	0	1	15
	-3	-2	0	0	0

 $x_1$  $x_2$  $u_1$  $u_2$  $u_1$ 

2

1

1

0

12

 $u_2$ 

1

3

0

1

15

-3

-2

0

0

0

 $x_1$  $x_2$  $u_1$  $u_2$  $\leftarrow u_1$ 

2

1

1

0

12

 $u_2$ 

1

3

0

1

15

-3

-2

0

0

0

 $x_1$  $x_2$  $u_1$  $u_2$  $u_2$ 

1	1/2	1/2	0	6
1	3	0	1	15
-3	-2	0	0	0

	$x_1$	$x_2$	$u_1$	$u_2$	
$x_1$	1	$1/2$	$1/2$	0	6
$u_2$	0	$5/2$	$-1/2$	1	9
	0	$-1/2$	$3/2$	0	18



	$x_1$	$x_2$	$u_1$	$u_2$	
$x_1$	1	1/2	1/2	0	6
$u_2$	0	5/2	-1/2	1	9
	0	-1/2	3/2	0	18





$x_1$       $x_2$       $u_1$       $u_2$

$x_1$

1	1/2	1/2	0	6
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$\leftarrow u_2$

0	5/2	-1/2	1	9
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0	-1/2	3/2	0	18
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$x_1$        $x_2$        $u_1$        $u_2$

$x_1$



1	1/2	1/2	0	6
0	1	-1/5	2/5	18/5
0	-1/2	3/2	0	18

	$x_1$	$x_2$	$u_1$	$u_2$	
$x_1$	1	0	$3/5$	$-1/5$	$21/5$
$x_2$	0	1	$-1/5$	$2/5$	$18/5$
	0	0	$7/5$	$1/5$	$99/5$

	$x_1$	$x_2$	$u_1$	$u_2$	
$x_1$	1	0	$3/5$	$-1/5$	$21/5$
$x_2$	0	1	$-1/5$	$2/5$	$18/5$
	0	0	$7/5$	$1/5$	$99/5$

$$x_2 - \frac{1}{5}u_1 + \frac{2}{5}u_2 = \frac{18}{5} \quad \text{and} \quad (x_1, x_2, u_1, u_2) \geq 0 \text{ in } \mathbb{Z}^4$$

$$x_2 + \left[-\frac{1}{5}\right]u_1 + \left[\frac{2}{5}\right]u_2 \leq \left[\frac{18}{5}\right]$$

$$x_2 - u_1 \leq 3$$

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	
$x_1$	1	0	$3/5$	$-1/5$	0	$21/5$
$x_2$	0	1	$-1/5$	$2/5$	0	$18/5$
$u_3$	0	1	-1	0	1	3
	0	0	$7/5$	$1/5$	0	$99/5$

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	
$x_1$	1	0	$3/5$	$-1/5$	0	$21/5$
$x_2$	0	1	$-1/5$	$2/5$	0	$18/5$
$u_3$	0	1	-1	0	1	3
	0	0	$7/5$	$1/5$	0	$99/5$

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	
$x_1$	1	0	$3/5$	$-1/5$	0	$21/5$
$x_2$	0	1	$-1/5$	$2/5$	0	$18/5$
$u_3$	0	0	$-4/5$	$-2/5$	1	$-3/5$
	0	0	$7/5$	$1/5$	0	$99/5$

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	
$x_1$	1	0	$3/5$	$-1/5$	0	$21/5$
$x_2$	0	1	$-1/5$	$2/5$	0	$18/5$
$\leftarrow u_3$	0	0	$-4/5$	$-2/5$	1	$-3/5$
	0	0	$7/5$	$1/5$	0	$99/5$



	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	
$x_1$	1	0	$3/5$	$-1/5$	0	$21/5$
$x_2$	0	1	$-1/5$	$2/5$	0	$18/5$
$u_3$	0	0	$-4/5$	$-2/5$	1	$-3/5$
	0	0	$7/5$	$1/5$	0	$99/5$

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	
$x_1$	1	0	$3/5$	$-1/5$	0	$21/5$
$x_2$	0	1	$-1/5$	$2/5$	0	$18/5$
	0	0	2	1	$-5/2$	$3/2$
	0	0	$7/5$	$1/5$	0	$99/5$

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	
$x_1$	1	0	1	0	-1/2	9/2
$x_2$	0	1	-1	0	1	3
$u_2$	0	0	2	1	-5/2	3/2
	0	0	1	0	1/2	39/2

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	
$x_1$	1	0	1	0	-1/2	9/2
$x_2$	0	1	-1	0	1	3
$u_2$	0	0	2	1	-5/2	3/2
	0	0	1	0	1/2	39/2

$$x_1 + u_1 - \frac{1}{2}u_3 = \frac{9}{2} \quad \text{and} \quad (x_1, x_2, u_1, u_2, u_3) \geq 0 \text{ in } \mathbb{Z}^5$$

$$x_1 + u_1 - u_3 \leq 4$$

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	
$x_1$	1	0	1	0	-1/2	0	9/2
$x_2$	0	1	-1	0	1	0	3
$u_2$	0	0	2	1	-5/2	0	3/2
$u_4$	1	0	1	0	-1	1	4
	0	0	1	0	1/2	0	39/2

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	
$x_1$	1	0	1	0	-1/2	0	9/2
$x_2$	0	1	-1	0	1	0	3
$u_2$	0	0	2	1	-5/2	0	3/2
$u_4$	1	0	1	0	-1	1	4
	0	0	1	0	1/2	0	39/2

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	
$x_1$	1	0	1	0	-1/2	0	9/2
$x_2$	0	1	-1	0	1	0	3
$u_2$	0	0	2	1	-5/2	0	3/2
$u_4$	0	0	0	0	-1/2	1	-1/2
	0	0	1	0	1/2	0	39/2

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	
$x_1$	1	0	1	0	-1/2	0	9/2
$x_2$	0	1	-1	0	1	0	3
$u_2$	0	0	2	1	-5/2	0	3/2
$\leftarrow u_4$	0	0	0	0	-1/2	1	-1/2
	0	0	1	0	1/2	0	39/2



	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	
$x_1$	1	0	1	0	-1/2	0	9/2
$x_2$	0	1	-1	0	1	0	3
$u_2$	0	0	2	1	-5/2	0	3/2
$u_4$	0	0	0	0	-1/2	1	-1/2
	0	0	1	0	1/2	0	39/2

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	
$x_1$	1	0	1	0	-1/2	0	9/2
$x_2$	0	1	-1	0	1	0	3
$u_2$	0	0	2	1	-5/2	0	3/2
	0	0	0	0	1	-2	1
	0	0	1	0	1/2	0	39/2

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	
$x_1$	1	0	1	0	0	-1	5
$x_2$	0	1	-1	0	0	2	2
$u_2$	0	0	2	1	0	-5	4
$u_3$	0	0	0	0	1	-2	1
	0	0	1	0	0	1	19

	$x_1$	$x_2$	$u_1$	$u_2$	$u_3$	$u_4$	
$x_1$	1	0	1	0	0	-1	5
$x_2$	0	1	-1	0	0	2	2
$u_2$	0	0	2	1	0	-5	4
$u_3$	0	0	0	0	1	-2	1
	0	0	1	0	0	1	19

Optimal solution:  $(x_1, x_2) = (5, 2)$