

Integer Problem:

Maximize $z = 5x_1 + 6x_2$

subject to

$$10x_1 + 3x_2 \leq 52$$

$$2x_1 + 3x_2 \leq 18$$

$$(x_1, x_2) \geq 0 \text{ in } \mathbb{Z}^2$$

Canonical Form:

Maximize $z = 5x_1 + 6x_2$

subject to

$$10x_1 + 3x_2 + u_1 = 52$$

$$2x_1 + 3x_2 + u_2 = 18$$

$$(x_1, x_2, u_1, u_2) \geq 0 \text{ in } \mathbb{Z}^4$$

	x_1	x_2	u_1	u_2	
u_1	10	3	1	0	52
u_2	2	3	0	1	18
	-5	-6	0	0	0

 x_1 x_2 u_1 u_2 u_1

10

3

1

0

52

 u_2

2

3

0

1

18

-5

-6

0

0

0

 x_1 x_2 u_1 u_2 $\leftarrow u_1$

10	3	1	0	52
2	3	0	1	18
-5	-6	0	0	0

3

1

0

52

 u_2

2

3

0

1

18

-5

-6

0

0

0

\downarrow

	x_1	x_2	u_1	u_2	
\leftarrow	1	$3/10$	$1/10$	0	$26/5$
u_2	2	3	0	1	18
	-5	-6	0	0	0

	x_1	x_2	u_1	u_2	
x_1	1	$3/10$	$1/10$	0	$26/5$
u_2	0	$12/5$	$-1/5$	1	$38/5$
	0	$-9/2$	$1/2$	0	26



	x_1	x_2	u_1	u_2	
x_1	1	$3/10$	$1/10$	0	$26/5$
u_2	0	$12/5$	$-1/5$	1	$38/5$
	0	$-9/2$	$1/2$	0	26



x_1 x_2 u_1 u_2

x_1

1	3/10	1/10	0	26/5
0	12/5	-1/5	1	38/5
0	-9/2	1/2	0	26

$\leftarrow u_2$



x_1 x_2 u_1 u_2

x_1



1	$3/10$	$1/10$	0	$26/5$
0	1	$-1/12$	$5/12$	$19/6$
0	$-9/2$	$1/2$	0	26

	x_1	x_2	u_1	u_2	
x_1	1	0	$1/8$	$-1/8$	$17/4$
x_2	0	1	$-1/12$	$5/12$	$19/6$
	0	0	$1/8$	$15/8$	$161/4$

	x_1	x_2	u_1	u_2	
x_1	1	0	1/8	-1/8	17/4
x_2	0	1	-1/12	5/12	19/6
	0	0	1/8	15/8	161/4

$$x_1 + \frac{1}{8}u_1 - \frac{1}{8}u_2 = \frac{17}{4} \quad \text{and} \quad (x_1, x_2, u_1, u_2) \geq 0 \text{ in } \mathbb{Z}^4$$

$$x_1 + \left[\frac{1}{8}\right]u_1 + \left[-\frac{1}{8}\right]u_2 \leq \left[\frac{17}{4}\right]$$

$$x_1 - u_2 \leq 4$$

	x_1	x_2	u_1	u_2	u_3	
x_1	1	0	1/8	-1/8	0	17/4
x_2	0	1	-1/12	5/12	0	19/6
u_3	1	0	0	-1	1	4
	0	0	1/8	15/8	0	161/4

	x_1	x_2	u_1	u_2	u_3	
x_1	1	0	$1/8$	$-1/8$	0	$17/4$
x_2	0	1	$-1/12$	$5/12$	0	$19/6$
u_3	1	0	0	-1	1	4
	0	0	$1/8$	$15/8$	0	$161/4$

	x_1	x_2	u_1	u_2	u_3	
x_1	1	0	$1/8$	$-1/8$	0	$17/4$
x_2	0	1	$-1/12$	$5/12$	0	$19/6$
u_3	0	0	$-1/8$	$-7/8$	1	$-1/4$
	0	0	$1/8$	$15/8$	0	$161/4$

	x_1	x_2	u_1	u_2	u_3	
x_1	1	0	$1/8$	$-1/8$	0	$17/4$
x_2	0	1	$-1/12$	$5/12$	0	$19/6$
$\leftarrow u_3$	0	0	$-1/8$	$-7/8$	1	$-1/4$
	0	0	$1/8$	$15/8$	0	$161/4$



x_1 x_2 u_1 u_2 u_3

x_1 1 0 1/8 -1/8 0 17/4

x_2 0 1 -1/12 5/12 0 19/6

$\leftarrow u_3$ 0 0 -1/8 -7/8 1 -1/4

0 0 1/8 15/8 0 161/4



x_1 x_2 u_1 u_2 u_3

x_1

1 0 1/8 -1/8 0 17/4

x_2

0 1 -1/12 5/12 0 19/6



0 0 1 7 -8 2

0 0 1/8 15/8 0 161/4

	x_1	x_2	u_1	u_2	u_3	
x_1	1	0	0	-1	1	4
x_2	0	1	0	1	-2/3	10/3
u_1	0	0	1	7	-8	2
	0	0	0	1	1	40

	x_1	x_2	u_1	u_2	u_3	
x_1	1	0	0	-1	1	4
x_2	0	1	0	1	-2/3	10/3
u_1	0	0	1	7	-8	2
	0	0	0	1	1	40

$$x_2 + u_2 - \frac{2}{3}u_3 = \frac{10}{3} \quad \text{and} \quad (x_1, x_2, u_1, u_2, u_3) \geq 0 \text{ in } \mathbb{Z}^5$$

$$x_2 + u_2 - u_3 \leq 3$$

	x_1	x_2	u_1	u_2	u_3	u_4	
x_1	1	0	0	-1	1	0	4
x_2	0	1	0	1	-2/3	0	10/3
u_1	0	0	1	7	-8	0	2
u_4	0	1	0	1	-1	1	3
	0	0	0	1	1	0	40

	x_1	x_2	u_1	u_2	u_3	u_4	
x_1	1	0	0	-1	1	0	4
x_2	0	1	0	1	-2/3	0	10/3
u_1	0	0	1	7	-8	0	2
u_4	0	1	0	1	-1	1	3
	0	0	0	1	1	0	40

	x_1	x_2	u_1	u_2	u_3	u_4	
x_1	1	0	0	-1	1	0	4
x_2	0	1	0	1	-2/3	0	10/3
u_1	0	0	1	7	-8	0	2
u_4	0	0	0	0	-1/3	1	-1/3
	0	0	0	1	1	0	40

	x_1	x_2	u_1	u_2	u_3	u_4	
x_1	1	0	0	-1	1	0	4
x_2	0	1	0	1	-2/3	0	10/3
u_1	0	0	1	7	-8	0	2
$\leftarrow u_4$	0	0	0	0	-1/3	1	-1/3
	0	0	0	1	1	0	40

	x_1	x_2	u_1	u_2	u_3	u_4	
x_1	1	0	0	-1	1	0	4
x_2	0	1	0	1	$-2/3$	0	$10/3$
u_1	0	0	1	7	-8	0	2
u_4	0	0	0	0	$-1/3$	1	$-1/3$
	0	0	0	1	1	0	40

	x_1	x_2	u_1	u_2	u_3	u_4	
x_1	1	0	0	-1	1	0	4
x_2	0	1	0	1	$-2/3$	0	$10/3$
u_1	0	0	1	7	-8	0	2
	0	0	0	0	1	-3	1
	0	0	0	1	1	0	40

A red arrow points down to the u_3 column header. A red arrow points left to the row containing the circled '1' in the u_3 column.

	x_1	x_2	u_1	u_2	u_3	u_4	
x_1	1	0	0	-1	0	3	3
x_2	0	1	0	1	0	-2	4
u_1	0	0	1	7	0	-24	10
u_3	0	0	0	0	1	-3	1
	0	0	0	1	0	3	39

	x_1	x_2	u_1	u_2	u_3	u_4	
x_1	1	0	0	-1	0	3	3
x_2	0	1	0	1	0	-2	4
u_1	0	0	1	7	0	-24	10
u_3	0	0	0	0	1	-3	1
	0	0	0	1	0	3	39

Optimal solution: $(x_1, x_2) = (3, 4)$