**Integer Problem:**

$$\text{Maximize } z = 3x_1 + 2x_2$$
$$\text{subject to}$$
$$2x_1 + \ x_2 \ \leq \ 12$$
$$\ x_1 + 3x_2 \ \leq \ 15$$
$$(x_1, x_2) \geq 0 \text{ in } \mathbb{Z}^2$$

**Canonical Form:**

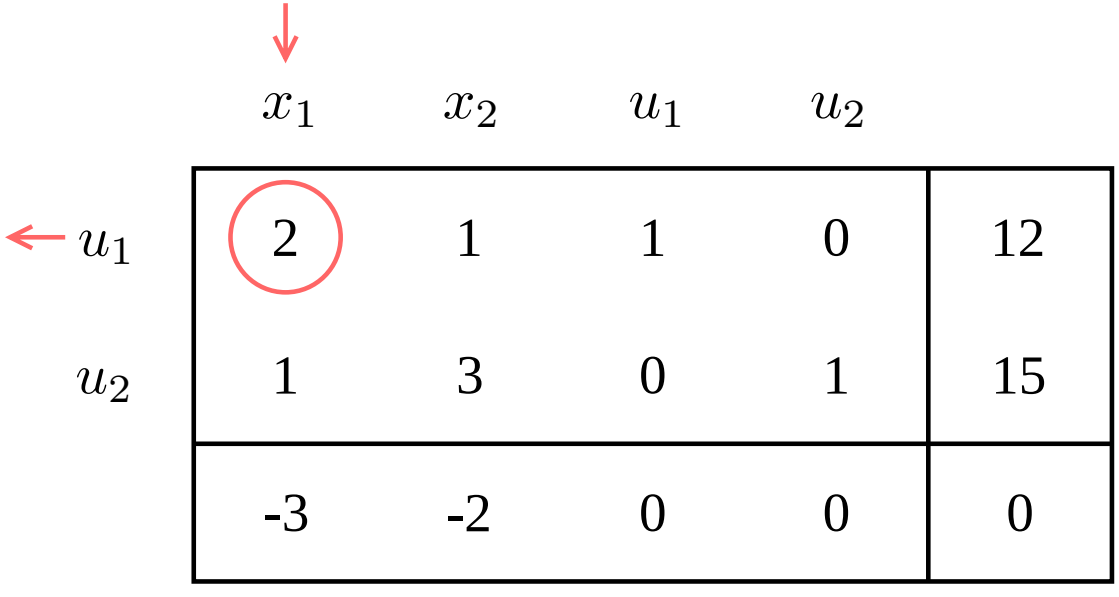
Maximize $z = 3x_1 + 2x_2$
subject to

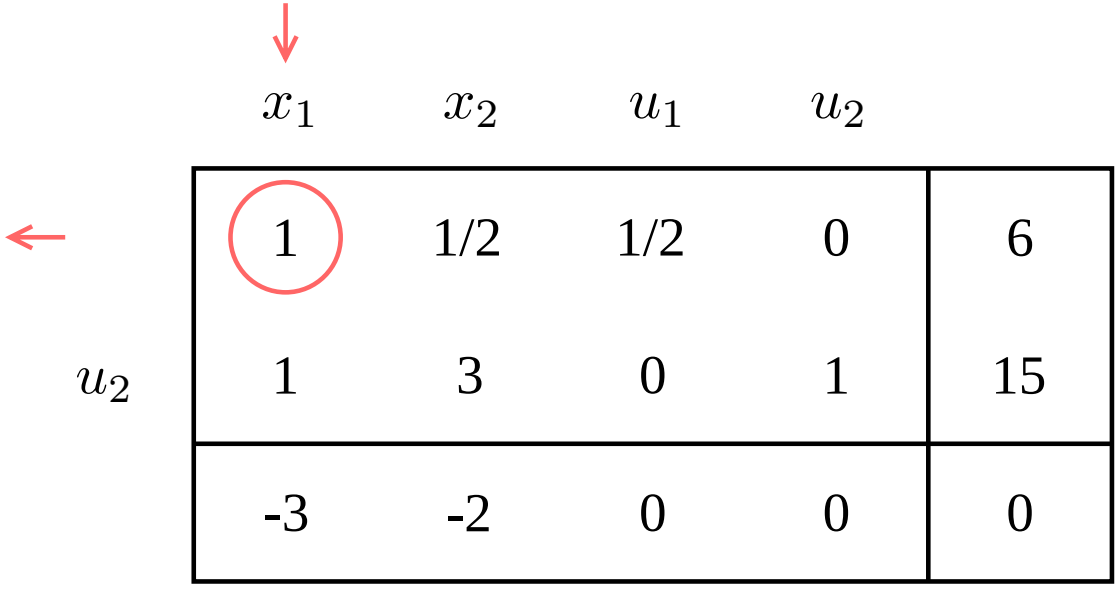
$$\begin{aligned} 2x_1 + \ \ x_2 + u_1 \qquad\quad &= \ 12 \\ \ x_1 + 3x_2 \qquad + u_2 \ &= \ 15 \end{aligned}$$

$(x_1, x_2, u_1, u_2) \geq 0$ in $\mathbb{Z}^4$

| | $x_1$ | $x_2$ | $u_1$ | $u_2$ | |
|---|---|---|---|---|---|
| $u_1$ | 2 | 1 | 1 | 0 | 12 |
| $u_2$ | 1 | 3 | 0 | 1 | 15 |
| | -3 | -2 | 0 | 0 | 0 |

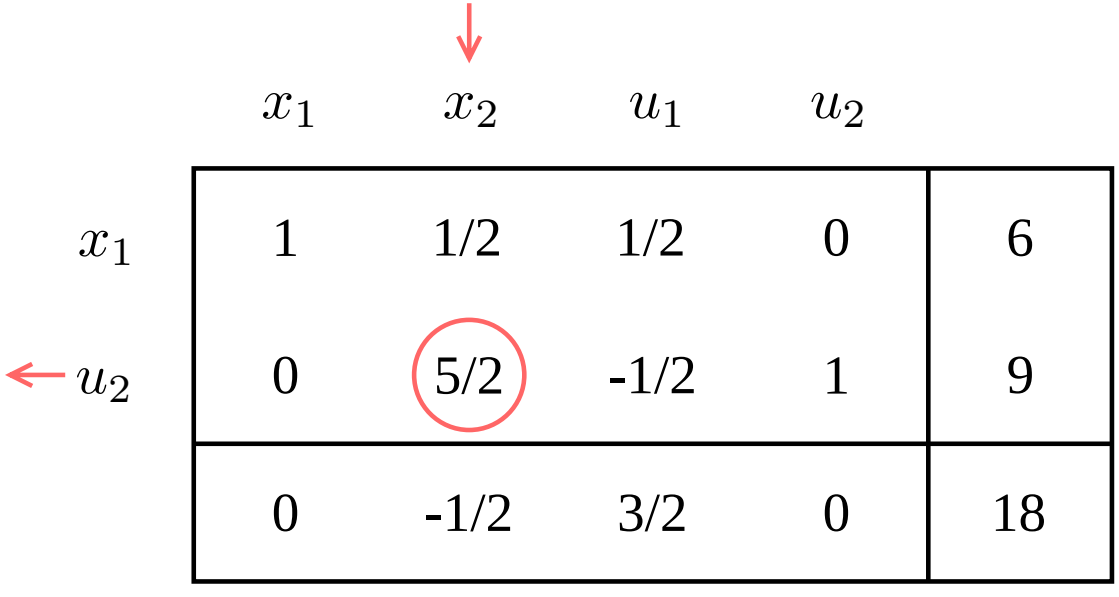
| | $x_1$ | $x_2$ | $u_1$ | $u_2$ | |
|---|---|---|---|---|---|
| $u_1$ | 2 | 1 | 1 | 0 | 12 |
| $u_2$ | 1 | 3 | 0 | 1 | 15 |
| | -3 | -2 | 0 | 0 | 0 |


$x_1$
$x_2$
$u_1$
$u_2$
$u_1$
$u_2$
2
1
1
0
12
1
3
0
1
15
-3
-2
0
0
0


$x_1$ $x_2$ $u_1$ $u_2$
1 1/2 1/2 0 6
$u_2$ 1 3 0 1 15
-3 -2 0 0 0

| | $x_1$ | $x_2$ | $u_1$ | $u_2$ | |
|---|---|---|---|---|---|
| $x_1$ | 1 | 1/2 | 1/2 | 0 | 6 |
| $u_2$ | 0 | 5/2 | -1/2 | 1 | 9 |
| | 0 | -1/2 | 3/2 | 0 | 18 |

| | $x_1$ | $x_2$ | $u_1$ | $u_2$ | |
|---|---|---|---|---|---|
| $x_1$ | 1 | 1/2 | 1/2 | 0 | 6 |
| $u_2$ | 0 | 5/2 | -1/2 | 1 | 9 |
| | 0 | -1/2 | 3/2 | 0 | 18 |


$x_1$
$x_2$
$u_1$
$u_2$
$x_1$
$u_2$
1
1/2
1/2
0
6
0
5/2
-1/2
1
9
0
-1/2
3/2
0
18

| | $x_1$ | $x_2$ | $u_1$ | $u_2$ | |
|---|---|---|---|---|---|
| $x_1$ | 1 | 1/2 | 1/2 | 0 | 6 |
| ← | 0 | 1 | -1/5 | 2/5 | 18/5 |
| | 0 | -1/2 | 3/2 | 0 | 18 |

| | $x_1$ | $x_2$ | $u_1$ | $u_2$ | |
|---|---|---|---|---|---|
| $x_1$ | 1 | 0 | 3/5 | -1/5 | 21/5 |
| $x_2$ | 0 | 1 | -1/5 | 2/5 | 18/5 |
| | 0 | 0 | 7/5 | 1/5 | 99/5 |

| | $x_1$ | $x_2$ | $u_1$ | $u_2$ | |
|---|---|---|---|---|---|
| $x_1$ | 1 | 0 | 3/5 | -1/5 | 21/5 |
| $x_2$ | 0 | 1 | -1/5 | 2/5 | 18/5 |
| | 0 | 0 | 7/5 | 1/5 | 99/5 |

$x_2 - \frac{1}{5}u_1 + \frac{2}{5}u_2 = \frac{18}{5}$ and $(x_1, x_2, u_1, u_2) \geq 0$ in $\mathbb{Z}^4$

Good cutting plane: $\frac{4}{5}u_1 + \frac{2}{5}u_2 \geq \frac{3}{5}$

Better cutting plane: $\frac{3}{10}u_1 + \frac{2}{5}u_2 \geq \frac{3}{5}$

**Revised mixed integer problem:**

Maximize $z = -7/5u_1 - 1/5u_2 + 99/5$
subject to

$$\begin{array}{rcl} x_1 + 3/5u_1 - 1/5u_2 & = & 21/5 \\ x_2 - 1/5u_1 + 2/5u_2 & = & 18/5 \\ -3/10u_1 - 2/5u_2 + u_3 & = & -3/5 \end{array}$$

$(x_1, x_2, u_1, u_2, u_3) \geq 0 \quad ; \quad x_1, x_2, u_1, u_2 \in \mathbb{Z} \quad ; \quad u_3 \in \mathbb{R}$

| | $x_1$ | $x_2$ | $u_1$ | $u_2$ | $u_3$ | |
|---|---|---|---|---|---|---|
| $x_1$ | 1 | 0 | 3/5 | -1/5 | 0 | 21/5 |
| $x_2$ | 0 | 1 | -1/5 | 2/5 | 0 | 18/5 |
| $u_3$ | 0 | 0 | -3/10 | -2/5 | 1 | -3/5 |
| | 0 | 0 | 7/5 | 1/5 | 0 | 99/5 |

| | $x_1$ | $x_2$ | $u_1$ | $u_2$ | $u_3$ | |
|---|---|---|---|---|---|---|
| $x_1$ | 1 | 0 | 3/5 | -1/5 | 0 | 21/5 |
| $x_2$ | 0 | 1 | -1/5 | 2/5 | 0 | 18/5 |
| $u_3$ | 0 | 0 | -3/10 | -2/5 | 1 | -3/5 |
| | 0 | 0 | 7/5 | 1/5 | 0 | 99/5 |

| | $x_1$ | $x_2$ | $u_1$ | $u_2$ | $u_3$ | |
|---|---|---|---|---|---|---|
| $x_1$ | 1 | 0 | 3/5 | -1/5 | 0 | 21/5 |
| $x_2$ | 0 | 1 | -1/5 | 2/5 | 0 | 18/5 |
| ← $u_3$ | 0 | 0 | -3/10 | -2/5 | 1 | -3/5 |
| | 0 | 0 | 7/5 | 1/5 | 0 | 99/5 |

| | $x_1$ | $x_2$ | $u_1$ | $u_2$ | $u_3$ | |
|---|---|---|---|---|---|---|
| $x_1$ | 1 | 0 | 3/5 | -1/5 | 0 | 21/5 |
| $x_2$ | 0 | 1 | -1/5 | 2/5 | 0 | 18/5 |
| ← $u_3$ | 0 | 0 | -3/10 | -2/5 | 1 | -3/5 |
| | 0 | 0 | 7/5 | 1/5 | 0 | 99/5 |

| | $x_1$ | $x_2$ | $u_1$ | $u_2$ | $u_3$ | |
|---|---|---|---|---|---|---|
| $x_1$ | 1 | 0 | 3/5 | -1/5 | 0 | 21/5 |
| $x_2$ | 0 | 1 | -1/5 | 2/5 | 0 | 18/5 |
| ← | 0 | 0 | 3/4 | 1 | -5/2 | 3/2 |
| | 0 | 0 | 7/5 | 1/5 | 0 | 99/5 |

| | $x_1$ | $x_2$ | $u_1$ | $u_2$ | $u_3$ | |
|---|---|---|---|---|---|---|
| $x_1$ | 1 | 0 | 3/4 | 0 | -1/2 | 9/2 |
| $x_2$ | 0 | 1 | -1/2 | 0 | 1 | 3 |
| $u_2$ | 0 | 0 | 3/4 | 1 | -5/2 | 3/2 |
| | 0 | 0 | 5/4 | 0 | 1/2 | 39/2 |

| | $x_1$ | $x_2$ | $u_1$ | $u_2$ | $u_3$ | |
|---|---|---|---|---|---|---|
| $x_1$ | 1 | 0 | 3/4 | 0 | -1/2 | 9/2 |
| $x_2$ | 0 | 1 | -1/2 | 0 | 1 | 3 |
| $u_2$ | 0 | 0 | 3/4 | 1 | -5/2 | 3/2 |
| | 0 | 0 | 5/4 | 0 | 1/2 | 39/2 |

$x_1 + \frac{3}{4}u_1 - \frac{1}{2}u_3 = \frac{9}{2}$ ; $(x_1, x_2, u_1, u_2) \geq 0$ in $\mathbb{Z}^4$ ; $u_3 \geq 0$ in $\mathbb{R}$

Mixed integer cutting plane: $\frac{1}{4}u_1 + \frac{1}{2}u_3 \geq \frac{1}{2}$

**Revised mixed integer problem:**

Maximize $z = -5/4u_1 - 1/2u_3 + 39/2$
subject to

$$\begin{array}{rrrrrcr}
x_1 & +3/4u_1 & & -1/2u_3 & & = & 9/2 \\
& x_2 - 1/2u_1 & & + & u_3 & = & 3 \\
& & 3/4u_1 + u_2 & - 5/2u_3 & & = & 3/2 \\
& & -1/4u_1 & - 1/2u_3 + u_4 & & = & -1/2
\end{array}$$

$(x_1, x_2, u_1, u_2, u_3, u_4) \geq 0 \quad ; \quad x_1, x_2, u_1, u_2 \in \mathbb{Z} \quad ; \quad u_3, u_4 \in \mathbb{R}$

| | $x_1$ | $x_2$ | $u_1$ | $u_2$ | $u_3$ | $u_4$ | |
|---|---|---|---|---|---|---|---|
| $x_1$ | 1 | 0 | 3/4 | 0 | -1/2 | 0 | 9/2 |
| $x_2$ | 0 | 1 | -1/2 | 0 | 1 | 0 | 3 |
| $u_2$ | 0 | 0 | 3/4 | 1 | -5/2 | 0 | 3/2 |
| $u_4$ | 0 | 0 | -1/4 | 0 | -1/2 | 1 | -1/2 |
| | 0 | 0 | 5/4 | 0 | 1/2 | 0 | 39/2 |

| | $x_1$ | $x_2$ | $u_1$ | $u_2$ | $u_3$ | $u_4$ | |
|---|---|---|---|---|---|---|---|
| $x_1$ | 1 | 0 | 3/4 | 0 | -1/2 | 0 | 9/2 |
| $x_2$ | 0 | 1 | -1/2 | 0 | 1 | 0 | 3 |
| $u_2$ | 0 | 0 | 3/4 | 1 | -5/2 | 0 | 3/2 |
| $u_4$ | 0 | 0 | -1/4 | 0 | -1/2 | 1 | -1/2 |
| | 0 | 0 | 5/4 | 0 | 1/2 | 0 | 39/2 |

| | $x_1$ | $x_2$ | $u_1$ | $u_2$ | $u_3$ | $u_4$ | |
|---|---|---|---|---|---|---|---|
| $x_1$ | 1 | 0 | 3/4 | 0 | -1/2 | 0 | 9/2 |
| $x_2$ | 0 | 1 | -1/2 | 0 | 1 | 0 | 3 |
| $u_2$ | 0 | 0 | 3/4 | 1 | -5/2 | 0 | 3/2 |
| ← $u_4$ | 0 | 0 | -1/4 | 0 | -1/2 | 1 | -1/2 |
| | 0 | 0 | 5/4 | 0 | 1/2 | 0 | 39/2 |

| | $x_1$ | $x_2$ | $u_1$ | $u_2$ | $u_3$ | $u_4$ | |
|---|---|---|---|---|---|---|---|
| $x_1$ | 1 | 0 | 3/4 | 0 | -1/2 | 0 | 9/2 |
| $x_2$ | 0 | 1 | -1/2 | 0 | 1 | 0 | 3 |
| $u_2$ | 0 | 0 | 3/4 | 1 | -5/2 | 0 | 3/2 |
| ← $u_4$ | 0 | 0 | -1/4 | 0 | -1/2 | 1 | -1/2 |
| | 0 | 0 | 5/4 | 0 | 1/2 | 0 | 39/2 |

| | $x_1$ | $x_2$ | $u_1$ | $u_2$ | $u_3$ | $u_4$ | |
|---|---|---|---|---|---|---|---|
| $x_1$ | 1 | 0 | 3/4 | 0 | -1/2 | 0 | 9/2 |
| $x_2$ | 0 | 1 | -1/2 | 0 | 1 | 0 | 3 |
| $u_2$ | 0 | 0 | 3/4 | 1 | -5/2 | 0 | 3/2 |
| ← | 0 | 0 | 1/2 | 0 | 1 | -2 | 1 |
| | 0 | 0 | 5/4 | 0 | 1/2 | 0 | 39/2 |

| | $x_1$ | $x_2$ | $u_1$ | $u_2$ | $u_3$ | $u_4$ | |
|---|---|---|---|---|---|---|---|
| $x_1$ | 1 | 0 | 1 | 0 | 0 | -1 | 5 |
| $x_2$ | 0 | 1 | -1 | 0 | 0 | 2 | 2 |
| $u_2$ | 0 | 0 | 2 | 1 | 0 | -5 | 4 |
| $u_3$ | 0 | 0 | 1/2 | 0 | 1 | -2 | 1 |
| | 0 | 0 | 1 | 0 | 0 | 1 | 19 |

| | $x_1$ | $x_2$ | $u_1$ | $u_2$ | $u_3$ | $u_4$ | |
|---|---|---|---|---|---|---|---|
| $x_1$ | 1 | 0 | 1 | 0 | 0 | -1 | 5 |
| $x_2$ | 0 | 1 | -1 | 0 | 0 | 2 | 2 |
| $u_2$ | 0 | 0 | 2 | 1 | 0 | -5 | 4 |
| $u_3$ | 0 | 0 | 1/2 | 0 | 1 | -2 | 1 |
| | 0 | 0 | 1 | 0 | 0 | 1 | 19 |

Optimal solution: $(x_1, x_2) = (5, 2)$