

### Mixed Integer Problem:

Maximize  $z = x_1 + 2x_2 + x_3 + x_4$

subject to

$$2x_1 + x_2 + 3x_3 + x_4 \leq 8$$

$$2x_1 + 3x_2 + 4x_4 \leq 12$$

$$3x_1 + x_2 + 2x_3 \leq 18$$

$$(x_1, x_2, x_3, x_4) \geq 0 \quad ; \quad x_2, x_4 \in \mathbb{R} \quad ; \quad x_1, x_3 \in \mathbb{Z}$$

### Canonical Form:

Maximize  $z = x_1 + 2x_2 + x_3 + x_4$

subject to

$$2x_1 + x_2 + 3x_3 + x_4 + u_1 = 8$$

$$2x_1 + 3x_2 + 4x_4 + u_2 = 12$$

$$3x_1 + x_2 + 2x_3 + u_3 = 18$$

$$(x_1, x_2, x_3, x_4, u_1, u_2, u_3) \geq 0 \text{ in } \mathbb{R}^7 ; \quad x_1, x_3 \in \mathbb{Z}$$

	$x_1$	$x_2$	$x_3$	$x_4$	$u_1$	$u_2$	$u_3$	
$u_1$	2	1	3	1	1	0	0	8
$u_2$	2	3	0	4	0	1	0	12
$u_3$	3	1	2	0	0	0	1	18
	-1	-2	-1	-1	0	0	0	0



	$x_1$	$x_2$	$x_3$	$x_4$	$u_1$	$u_2$	$u_3$	
$u_1$	2	1	3	1	1	0	0	8
$u_2$	2	3	0	4	0	1	0	12
$u_3$	3	1	2	0	0	0	1	18
	-1	-2	-1	-1	0	0	0	0



	$x_1$	$x_2$	$x_3$	$x_4$	$u_1$	$u_2$	$u_3$	
$u_1$	2	1	3	1	1	0	0	8
$u_2$	2	3	0	4	0	1	0	12
$u_3$	3	1	2	0	0	0	1	18
	-1	-2	-1	-1	0	0	0	0



	$x_1$	$x_2$	$x_3$	$x_4$	$u_1$	$u_2$	$u_3$	
$u_1$	2	1	3	1	1	0	0	8
$u_3$	2/3	1	0	4/3	0	1/3	0	4
	3	1	2	0	0	0	1	18
	-1	-2	-1	-1	0	0	0	0

	$x_1$	$x_2$	$x_3$	$x_4$	$u_1$	$u_2$	$u_3$	
$u_1$	$4/3$	$0$	$3$	$-1/3$	$1$	$-1/3$	$0$	$4$
$x_2$	$2/3$	$1$	$0$	$4/3$	$0$	$1/3$	$0$	$4$
$u_3$	$7/3$	$0$	$2$	$-4/3$	$0$	$-1/3$	$1$	$14$
	$1/3$	$0$	$-1$	$5/3$	$0$	$2/3$	$0$	$8$



	$x_1$	$x_2$	$x_3$	$x_4$	$u_1$	$u_2$	$u_3$	
$u_1$	$4/3$	$0$	$3$	$-1/3$	$1$	$-1/3$	$0$	$4$
$x_2$	$2/3$	$1$	$0$	$4/3$	$0$	$1/3$	$0$	$4$
$u_3$	$7/3$	$0$	$2$	$-4/3$	$0$	$-1/3$	$1$	$14$
	$1/3$	$0$	$-1$	$5/3$	$0$	$2/3$	$0$	$8$



	$x_1$	$x_2$	$x_3$	$x_4$	$u_1$	$u_2$	$u_3$	
$\leftarrow u_1$	$4/3$	$0$	$3$	$-1/3$	$1$	$-1/3$	$0$	$4$
$x_2$	$2/3$	$1$	$0$	$4/3$	$0$	$1/3$	$0$	$4$
$u_3$	$7/3$	$0$	$2$	$-4/3$	$0$	$-1/3$	$1$	$14$
	$1/3$	$0$	$-1$	$5/3$	$0$	$2/3$	$0$	$8$

	$x_1$	$x_2$	$x_3$	$x_4$	$u_1$	$u_2$	$u_3$	
$\leftarrow$	$4/9$	$0$	$1$	$-1/9$	$1/3$	$-1/9$	$0$	$4/3$
$x_2$	$2/3$	$1$	$0$	$4/3$	$0$	$1/3$	$0$	$4$
$u_3$	$7/3$	$0$	$2$	$-4/3$	$0$	$-1/3$	$1$	$14$
	$1/3$	$0$	$-1$	$5/3$	$0$	$2/3$	$0$	$8$

	$x_1$	$x_2$	$x_3$	$x_4$	$u_1$	$u_2$	$u_3$	
$x_3$	$4/9$	$0$	$1$	$-1/9$	$1/3$	$-1/9$	$0$	$4/3$
$x_2$	$2/3$	$1$	$0$	$4/3$	$0$	$1/3$	$0$	$4$
$u_3$	$13/9$	$0$	$0$	$-10/9$	$-2/3$	$-1/9$	$1$	$34/3$
	$7/9$	$0$	$0$	$14/9$	$1/3$	$5/9$	$0$	$28/3$

	$x_1$	$x_2$	$x_3$	$x_4$	$u_1$	$u_2$	$u_3$	
$x_3$	4/9	0	1	-1/9	1/3	-1/9	0	4/3
$x_2$	2/3	1	0	4/3	0	1/3	0	4
$u_3$	13/9	0	0	-10/9	-2/3	-1/9	1	34/3
	7/9	0	0	14/9	1/3	5/9	0	28/3

$$\frac{4}{9}x_1 + x_3 - \frac{1}{9}x_4 + \frac{1}{3}u_1 - \frac{1}{9}u_2 = \frac{4}{3} \quad ; \quad (x, u) \geq 0 \quad ; \quad x_1, x_3 \in \mathbb{Z}$$

$$\text{Mixed integer cutting plane: } \frac{5}{18}x_1 + \frac{1}{18}x_4 + \frac{1}{3}u_1 + \frac{1}{18}u_2 \geq \frac{1}{3}$$

	$x_1$	$x_2$	$x_3$	$x_4$	$u_1$	$u_2$	$u_3$	$u_4$	
$x_3$	$4/9$	$0$	$1$	$-1/9$	$1/3$	$-1/9$	$0$	$0$	$4/3$
$x_2$	$2/3$	$1$	$0$	$4/3$	$0$	$1/3$	$0$	$0$	$4$
$u_3$	$13/9$	$0$	$0$	$-10/9$	$-2/3$	$-1/9$	$1$	$0$	$34/3$
$u_4$	$-5/18$	$0$	$0$	$-1/18$	$-1/3$	$-1/18$	$0$	$1$	$-1/3$
	$7/9$	$0$	$0$	$14/9$	$1/3$	$5/9$	$0$	$0$	$28/3$

	$x_1$	$x_2$	$x_3$	$x_4$	$u_1$	$u_2$	$u_3$	$u_4$	
$x_3$	4/9	0	1	-1/9	1/3	-1/9	0	0	4/3
$x_2$	2/3	1	0	4/3	0	1/3	0	0	4
$u_3$	13/9	0	0	-10/9	-2/3	-1/9	1	0	34/3
$u_4$	-5/18	0	0	-1/18	-1/3	-1/18	0	1	-1/3
	7/9	0	0	14/9	1/3	5/9	0	0	28/3

	$x_1$	$x_2$	$x_3$	$x_4$	$u_1$	$u_2$	$u_3$	$u_4$	
$x_3$	$4/9$	$0$	$1$	$-1/9$	$1/3$	$-1/9$	$0$	$0$	$4/3$
$x_2$	$2/3$	$1$	$0$	$4/3$	$0$	$1/3$	$0$	$0$	$4$
$u_3$	$13/9$	$0$	$0$	$-10/9$	$-2/3$	$-1/9$	$1$	$0$	$34/3$
$\leftarrow u_4$	$-5/18$	$0$	$0$	$-1/18$	$-1/3$	$-1/18$	$0$	$1$	$-1/3$
	$7/9$	$0$	$0$	$14/9$	$1/3$	$5/9$	$0$	$0$	$28/3$

	$x_1$	$x_2$	$x_3$	$x_4$	$u_1$	$u_2$	$u_3$	$u_4$	
$x_3$	$4/9$	$0$	$1$	$-1/9$	$1/3$	$-1/9$	$0$	$0$	$4/3$
$x_2$	$2/3$	$1$	$0$	$4/3$	$0$	$1/3$	$0$	$0$	$4$
$u_3$	$13/9$	$0$	$0$	$-10/9$	$-2/3$	$-1/9$	$1$	$0$	$34/3$
$u_4$	$-5/18$	$0$	$0$	$-1/18$	$-1/3$	$-1/18$	$0$	$1$	$-1/3$
	$7/9$	$0$	$0$	$14/9$	$1/3$	$5/9$	$0$	$0$	$28/3$



	$x_1$	$x_2$	$x_3$	$x_4$	$u_1$	$u_2$	$u_3$	$u_4$	
$x_3$	$4/9$	$0$	$1$	$-1/9$	$1/3$	$-1/9$	$0$	$0$	$4/3$
$x_2$	$2/3$	$1$	$0$	$4/3$	$0$	$1/3$	$0$	$0$	$4$
$u_3$	$13/9$	$0$	$0$	$-10/9$	$-2/3$	$-1/9$	$1$	$0$	$34/3$
$\leftarrow$	$5/6$	$0$	$0$	$1/6$	$1$	$1/6$	$0$	$-3$	$1$
	$7/9$	$0$	$0$	$14/9$	$1/3$	$5/9$	$0$	$0$	$28/3$

	$x_1$	$x_2$	$x_3$	$x_4$	$u_1$	$u_2$	$u_3$	$u_4$	
$x_3$	1/6	0	1	-1/6	0	-1/6	0	1	1
$x_2$	2/3	1	0	4/3	0	1/3	0	0	4
$u_3$	2	0	0	-1	0	0	1	-2	12
$u_1$	5/6	0	0	1/6	1	1/6	0	-3	1
	1/2	0	0	3/2	0	1/2	0	1	9

	$x_1$	$x_2$	$x_3$	$x_4$	$u_1$	$u_2$	$u_3$	$u_4$	
$x_3$	1/6	0	1	-1/6	0	-1/6	0	1	1
$x_2$	2/3	1	0	4/3	0	1/3	0	0	4
$u_3$	2	0	0	-1	0	0	1	-2	12
$u_1$	5/6	0	0	1/6	1	1/6	0	-3	1
	1/2	0	0	3/2	0	1/2	0	1	9

Optimal solution:  $(x_1, x_2, x_3, x_4) = (0, 4, 1, 0)$