

Mixed Integer Problem:

Maximize $z = 11x_1 + 80x_2 + 13x_3$

subject to

$$3x_1 + 4x_2 + x_3 \leq 20$$

$$x_1 + 10x_2 + x_3 \leq 22$$

$$x_1 + 8x_2 + 3x_3 \leq 28$$

$$x \geq 0 \text{ in } \mathbb{R}^3 ; \quad x_1, x_3 \in \mathbb{Z}$$

Canonical Form:

Maximize $z = 11x_1 + 80x_2 + 13x_3$

subject to


$$3x_1 + 4x_2 + x_3 + u_1 = 20$$

$$x_1 + 10x_2 + x_3 + u_2 = 22$$

$$x_1 + 8x_2 + 3x_3 + u_3 = 28$$

$$(x, u) \geq 0 \text{ in } \mathbb{R}^6 ; \quad x_1, x_3 \in \mathbb{Z}$$

	x_1	x_2	x_3	u_1	u_2	u_3	
u_1	3	4	1	1	0	0	20
u_2	1	10	1	0	1	0	22
u_3	1	8	3	0	0	1	28
	-11	-80	-13	0	0	0	0

	x_1	 x_2	x_3	u_1	u_2	u_3	
u_1	3	4	1	1	0	0	20
u_2	1	10	1	0	1	0	22
u_3	1	8	3	0	0	1	28
	-11	-80	-13	0	0	0	0

	x_1	x_2	x_3	u_1	u_2	u_3	
u_1	3	4	1	1	0	0	20
u_2	1	10	1	0	1	0	22
u_3	1	8	3	0	0	1	28
	-11	-80	-13	0	0	0	0

	x_1	x_2	x_3	u_1	u_2	u_3	
u_1	3	4	1	1	0	0	20
u_2	1/10	1	1/10	0	1/10	0	11/5
u_3	1	8	3	0	0	1	28
	-11	-80	-13	0	0	0	0


	x_1	x_2	x_3	u_1	u_2	u_3	
u_1	13/5	0	3/5	1	-2/5	0	56/5
x_2	1/10	1	1/10	0	1/10	0	11/5
u_3	1/5	0	11/5	0	-4/5	1	52/5
	-3	0	-5	0	8	0	176

	x_1	x_2	x_3	u_1	u_2	u_3	
u_1	$13/5$	0	$3/5$	1	$-2/5$	0	$56/5$
x_2	$1/10$	1	$1/10$	0	$1/10$	0	$11/5$
u_3	$1/5$	0	$11/5$	0	$-4/5$	1	$52/5$
	-3	0	-5	0	8	0	176

	x_1	x_2	x_3	u_1	u_2	u_3	
u_1	$13/5$	0	$3/5$	1	$-2/5$	0	$56/5$
x_2	$1/10$	1	$1/10$	0	$1/10$	0	$11/5$
$\leftarrow u_3$	$1/5$	0	$11/5$	0	$-4/5$	1	$52/5$
	-3	0	-5	0	8	0	176

	x_1	x_2	x_3	u_1	u_2	u_3	
u_1	13/5	0	3/5	1	-2/5	0	56/5
x_2	1/10	1	1/10	0	1/10	0	11/5
	1/11	0	1	0	-4/11	5/11	52/11
	-3	0	-5	0	8	0	176

	x_1	x_2	x_3	u_1	u_2	u_3	
u_1	28/11	0	0	1	-2/11	-3/11	92/11
x_2	1/11	1	0	0	3/22	-1/22	19/11
x_3	1/11	0	1	0	-4/11	5/11	52/11
	-28/11	0	0	0	68/11	25/11	2196/11

	 x_1	x_2	x_3	u_1	u_2	u_3	
u_1	28/11	0	0	1	-2/11	-3/11	92/11
x_2	1/11	1	0	0	3/22	-1/22	19/11
x_3	1/11	0	1	0	-4/11	5/11	52/11
	-28/11	0	0	0	68/11	25/11	2196/11

	x_1	x_2	x_3	u_1	u_2	u_3	
u_1	$28/11$	0	0	1	$-2/11$	$-3/11$	$92/11$
x_2	$1/11$	1	0	0	$3/22$	$-1/22$	$19/11$
x_3	$1/11$	0	1	0	$-4/11$	$5/11$	$52/11$
	$-28/11$	0	0	0	$68/11$	$25/11$	$2196/11$

	x_1	x_2	x_3	u_1	u_2	u_3	
x_2	1	0	0	11/28	-1/14	-3/28	23/7
x_3	1/11	1	0	0	3/22	-1/22	19/11
	1/11	0	1	0	-4/11	5/11	52/11
	-28/11	0	0	0	68/11	25/11	2196/11

	x_1	x_2	x_3	u_1	u_2	u_3	
x_1	1	0	0	$11/28$	$-1/14$	$-3/28$	$23/7$
x_2	0	1	0	$-1/28$	$1/7$	$-1/28$	$10/7$
x_3	0	0	1	$-1/28$	$-5/14$	$13/28$	$31/7$
	0	0	0	1	6	2	208

	x_1	x_2	x_3	u_1	u_2	u_3	
x_1	1	0	0	11/28	-1/14	-3/28	23/7
x_2	0	1	0	-1/28	1/7	-1/28	10/7
x_3	0	0	1	-1/28	-5/14	13/28	31/7
	0	0	0	1	6	2	208

Current BFS: $(23/7, 10/7, 31/7) = (3.285714, 1.428571, 4.428571)$

	x_1	x_2	x_3	u_1	u_2	u_3	
x_1	1	0	0	11/28	-1/14	-3/28	23/7
x_2	0	1	0	-1/28	1/7	-1/28	10/7
x_3	0	0	1	-1/28	-5/14	13/28	31/7
	0	0	0	1	6	2	208

$$x_1 + \frac{11}{28}u_1 - \frac{1}{14}u_2 - \frac{3}{28}u_3 = \frac{23}{7} \quad ; \quad (x, u) \geq 0 \text{ in } \mathbb{R}^6 \quad ; \quad x_1, x_3 \in \mathbb{Z}$$

$$\text{Mixed integer cutting plane: } \frac{11}{28}u_1 + \frac{1}{35}u_2 + \frac{3}{70}u_3 \geq \frac{2}{7}$$

	x_1	x_2	x_3	u_1	u_2	u_3	u_4	
x_1	1	0	0	11/28	-1/14	-3/28	0	23/7
x_2	0	1	0	-1/28	1/7	-1/28	0	10/7
x_3	0	0	1	-1/28	-5/14	13/28	0	31/7
u_4	0	0	0	-11/28	-1/35	-3/70	1	-2/7
	0	0	0	1	6	2	0	208

	x_1	x_2	x_3	u_1	u_2	u_3	u_4	
x_1	1	0	0	11/28	-1/14	-3/28	0	23/7
x_2	0	1	0	-1/28	1/7	-1/28	0	10/7
x_3	0	0	1	-1/28	-5/14	13/28	0	31/7
$\leftarrow u_4$	0	0	0	-11/28	-1/35	-3/70	1	-2/7
	0	0	0	1	6	2	0	208



	x_1	x_2	x_3	u_1	u_2	u_3	u_4	
x_1	1	0	0	11/28	-1/14	-3/28	0	23/7
x_2	0	1	0	-1/28	1/7	-1/28	0	10/7
x_3	0	0	1	-1/28	-5/14	13/28	0	31/7
u_4	0	0	0	-11/28	-1/35	-3/70	1	-2/7
	0	0	0	1	6	2	0	208



x_1 x_2 x_3 u_1 u_2 u_3 u_4

x_1	1	0	0	11/28	-1/14	-3/28	0	23/7
x_2	0	1	0	-1/28	1/7	-1/28	0	10/7
x_3	0	0	1	-1/28	-5/14	13/28	0	31/7
	0	0	0	1	4/55	6/55	-28/11	8/11
	0	0	0	1	6	2	0	208



	x_1	x_2	x_3	u_1	u_2	u_3	u_4	
x_1	1	0	0	0	$-1/10$	$-3/20$	1	3
x_2	0	1	0	0	$8/55$	$-7/220$	$-1/11$	$16/11$
x_3	0	0	1	0	$-39/110$	$103/220$	$-1/11$	$49/11$
u_1	0	0	0	1	$4/55$	$6/55$	$-28/11$	$8/11$
	0	0	0	0	$326/55$	$104/55$	$28/11$	$2280/11$

	x_1	x_2	x_3	u_1	u_2	u_3	u_4	
x_1	1	0	0	0	-1/10	-3/20	1	3
x_2	0	1	0	0	8/55	-7/220	-1/11	16/11
x_3	0	0	1	0	-39/110	103/220	-1/11	49/11
u_1	0	0	0	1	4/55	6/55	-28/11	8/11
	0	0	0	0	326/55	104/55	28/11	2280/11

Current BFS: $(3, 16/11, 49/11) = (3.000000, 1.454545, 4.454545)$

	x_1	x_2	x_3	u_1	u_2	u_3	u_4	
x_1	1	0	0	0	-1/10	-3/20	1	3
x_2	0	1	0	0	8/55	-7/220	-1/11	16/11
x_3	0	0	1	0	-39/110	103/220	-1/11	49/11
u_1	0	0	0	1	4/55	6/55	-28/11	8/11
	0	0	0	0	326/55	104/55	28/11	2280/11

$$x_3 - \frac{39}{110}u_2 + \frac{103}{220}u_3 - \frac{1}{11}u_4 = \frac{49}{11} \quad ; \quad (x, u) \geq 0 \text{ in } \mathbb{R}^7 \quad ; \quad x_1, x_3 \in \mathbb{Z}$$

$$\text{Mixed integer cutting plane: } \frac{13}{44}u_2 + \frac{103}{220}u_3 + \frac{5}{66}u_4 \geq \frac{5}{11}$$

	x_1	x_2	x_3	u_1	u_2	u_3	u_4	u_5	
x_1	1	0	0	0	-1/10	-3/20	1	0	3
x_2	0	1	0	0	8/55	-7/220	-1/11	0	16/11
x_3	0	0	1	0	-39/110	103/220	-1/11	0	49/11
u_1	0	0	0	1	4/55	6/55	-28/11	0	8/11
u_5	0	0	0	0	-13/44	-103/220	-5/66	1	-5/11
	0	0	0	0	326/55	104/55	28/11	0	2280/11

	x_1	x_2	x_3	u_1	u_2	u_3	u_4	u_5	
x_1	1	0	0	0	-1/10	-3/20	1	0	3
x_2	0	1	0	0	8/55	-7/220	-1/11	0	16/11
x_3	0	0	1	0	-39/110	103/220	-1/11	0	49/11
u_1	0	0	0	1	4/55	6/55	-28/11	0	8/11
$\leftarrow u_5$	0	0	0	0	-13/44	-103/220	-5/66	1	-5/11
	0	0	0	0	326/55	104/55	28/11	0	2280/11

	x_1	x_2	x_3	u_1	u_2	u_3	u_4	u_5	
x_1	1	0	0	0	-1/10	-3/20	1	0	3
x_2	0	1	0	0	8/55	-7/220	-1/11	0	16/11
x_3	0	0	1	0	-39/110	103/220	-1/11	0	49/11
u_1	0	0	0	1	4/55	6/55	-28/11	0	8/11
u_5	0	0	0	0	-13/44	-103/220	-5/66	1	-5/11
	0	0	0	0	326/55	104/55	28/11	0	2280/11

	x_1	x_2	x_3	u_1	u_2	u_3	u_4	u_5	
x_1	1	0	0	0	-1/10	-3/20	1	0	3
x_2	0	1	0	0	8/55	-7/220	-1/11	0	16/11
x_3	0	0	1	0	-39/110	103/220	-1/11	0	49/11
u_1	0	0	0	1	4/55	6/55	-28/11	0	8/11
	0	0	0	0	65/103	1	50/309	-220/103	100/103
	0	0	0	0	326/55	104/55	28/11	0	2280/11

	x_1	x_2	x_3	u_1	u_2	u_3	u_4	u_5	
x_1	1	0	0	0	$-11/2060$	0	$211/206$	$-33/103$	$324/103$
x_2	0	1	0	0	$341/2060$	0	$-53/618$	$-7/103$	$153/103$
x_3	0	0	1	0	$-13/20$	0	$-1/6$	1	4
u_1	0	0	0	1	$2/515$	0	$-264/103$	$24/103$	$64/103$
u_3	0	0	0	0	$65/103$	1	$50/309$	$-220/103$	$100/103$
	0	0	0	0	$2438/515$	0	$692/309$	$416/103$	$21160/103$

	x_1	x_2	x_3	u_1	u_2	u_3	u_4	u_5	
x_1	1	0	0	0	-11/2060	0	211/206	-33/103	324/103
x_2	0	1	0	0	341/2060	0	-53/618	-7/103	153/103
x_3	0	0	1	0	-13/20	0	-1/6	1	4
u_1	0	0	0	1	2/515	0	-264/103	24/103	64/103
u_3	0	0	0	0	65/103	1	50/309	-220/103	100/103
	0	0	0	0	2438/515	0	692/309	416/103	21160/103

Current BFS: $(324/103, 153/103, 4) = (3.145631, 1.485437, 4.000000)$

	x_1	x_2	x_3	u_1	u_2	u_3	u_4	u_5	
x_1	1	0	0	0	-11/2060	0	211/206	-33/103	324/103
x_2	0	1	0	0	341/2060	0	-53/618	-7/103	153/103
x_3	0	0	1	0	-13/20	0	-1/6	1	4
u_1	0	0	0	1	2/515	0	-264/103	24/103	64/103
u_3	0	0	0	0	65/103	1	50/309	-220/103	100/103
	0	0	0	0	2438/515	0	692/309	416/103	21160/103

$$x_1 - \frac{11}{2060}u_2 + \frac{211}{206}u_4 - \frac{33}{103}u_5 = \frac{324}{103} \quad ; \quad (x, u) \geq 0 \text{ in } \mathbb{R}^8 \quad ; \quad x_1, x_3 \in \mathbb{Z}$$

$$\text{Mixed integer cutting plane: } \frac{3}{3296}u_2 + \frac{211}{206}u_4 + \frac{45}{824}u_5 \geq \frac{15}{103}$$

	x_1	x_2	x_3	u_1	u_2	u_3	u_4	u_5	u_6	
x_1	1	0	0	0	-11/2060	0	211/206	-33/103	0	324/103
x_2	0	1	0	0	341/2060	0	-53/618	-7/103	0	153/103
x_3	0	0	1	0	-13/20	0	-1/6	1	0	4
u_1	0	0	0	1	2/515	0	-264/103	24/103	0	64/103
u_3	0	0	0	0	65/103	1	50/309	-220/103	0	100/103
u_6	0	0	0	0	-3/3296	0	-211/206	-45/824	1	-15/103
	0	0	0	0	2438/515	0	692/309	416/103	0	21160/103

	x_1	x_2	x_3	u_1	u_2	u_3	u_4	u_5	u_6	
x_1	1	0	0	0	$-11/2060$	0	$211/206$	$-33/103$	0	$324/103$
x_2	0	1	0	0	$341/2060$	0	$-53/618$	$-7/103$	0	$153/103$
x_3	0	0	1	0	$-13/20$	0	$-1/6$	1	0	4
u_1	0	0	0	1	$2/515$	0	$-264/103$	$24/103$	0	$64/103$
u_3	0	0	0	0	$65/103$	1	$50/309$	$-220/103$	0	$100/103$
$\leftarrow u_6$	0	0	0	0	$-3/3296$	0	$-211/206$	$-45/824$	1	$-15/103$
	0	0	0	0	$2438/515$	0	$692/309$	$416/103$	0	$21160/103$

	x_1	x_2	x_3	u_1	u_2	u_3	u_4	u_5	u_6	
x_1	1	0	0	0	$-11/2060$	0	$211/206$	$-33/103$	0	$324/103$
x_2	0	1	0	0	$341/2060$	0	$-53/618$	$-7/103$	0	$153/103$
x_3	0	0	1	0	$-13/20$	0	$-1/6$	1	0	4
u_1	0	0	0	1	$2/515$	0	$-264/103$	$24/103$	0	$64/103$
u_3	0	0	0	0	$65/103$	1	$50/309$	$-220/103$	0	$100/103$
u_6	0	0	0	0	$-3/3296$	0	$-211/206$	$-45/824$	1	$-15/103$
	0	0	0	0	$2438/515$	0	$692/309$	$416/103$	0	$21160/103$

	x_1	x_2	x_3	u_1	u_2	u_3	u_4	u_5	u_6	
x_1	1	0	0	0	$-11/2060$	0	$211/206$	$-33/103$	0	$324/103$
x_2	0	1	0	0	$341/2060$	0	$-53/618$	$-7/103$	0	$153/103$
x_3	0	0	1	0	$-13/20$	0	$-1/6$	1	0	4
u_1	0	0	0	1	$2/515$	0	$-264/103$	$24/103$	0	$64/103$
u_3	0	0	0	0	$65/103$	1	$50/309$	$-220/103$	0	$100/103$
	0	0	0	0	$3/3376$	0	1	$45/844$	$-206/211$	$30/211$
	0	0	0	0	$2438/515$	0	$692/309$	$416/103$	0	$21160/103$

	x_1	x_2	x_3	u_1	u_2	u_3	u_4	u_5	u_6	
x_1	1	0	0	0	-1/160	0	0	-3/8	1	3
x_2	0	1	0	0	5591/33760	0	0	-107/1688	-53/633	316/211
x_3	0	0	1	0	-21939/33760	0	0	1703/1688	-103/633	849/211
u_1	0	0	0	1	13/2110	0	0	78/211	-528/211	208/211
u_3	0	0	0	0	1065/1688	1	0	-905/422	100/633	200/211
u_4	0	0	0	0	3/3376	0	1	45/844	-206/211	30/211
	0	0	0	0	19969/4220	0	0	827/211	1384/633	43280/211

	x_1	x_2	x_3	u_1	u_2	u_3	u_4	u_5	u_6	
x_1	1	0	0	0	-1/160	0	0	-3/8	1	3
x_2	0	1	0	0	5591/33760	0	0	-107/1688	-53/633	316/211
x_3	0	0	1	0	-21939/33760	0	0	1703/1688	-103/633	849/211
u_1	0	0	0	1	13/2110	0	0	78/211	-528/211	208/211
u_3	0	0	0	0	1065/1688	1	0	-905/422	100/633	200/211
u_4	0	0	0	0	3/3376	0	1	45/844	-206/211	30/211
	0	0	0	0	19969/4220	0	0	827/211	1384/633	43280/211

Current BFS: $(3, 316/211, 849/211) = (3.000000, 1.497630, 4.023697)$

	x_1	x_2	x_3	u_1	u_2	u_3	u_4	u_5	u_6	
x_1	1	0	0	0	-1/160	0	0	-3/8	1	3
x_2	0	1	0	0	5591/33760	0	0	-107/1688	-53/633	316/211
x_3	0	0	1	0	-21939/33760	0	0	1703/1688	-103/633	849/211
u_1	0	0	0	1	13/2110	0	0	78/211	-528/211	208/211
u_3	0	0	0	0	1065/1688	1	0	-905/422	100/633	200/211
u_4	0	0	0	0	3/3376	0	1	45/844	-206/211	30/211
	0	0	0	0	19969/4220	0	0	827/211	1384/633	43280/211

$$x_3 - \frac{21939}{33760}u_2 + \frac{1703}{1688}u_5 - \frac{103}{633}u_6 = \frac{849}{211} \quad ; \quad (x, u) \geq 0 \text{ in } \mathbb{R}^9 \quad ; \quad x_1, x_3 \in \mathbb{Z}$$

$$\text{Mixed integer cutting plane: } \frac{213}{13504}u_2 + \frac{1703}{1688}u_5 + \frac{5}{1266}u_6 \geq \frac{5}{211}$$

	x_1	x_2	x_3	u_1	u_2	u_3	u_4	u_5	u_6	u_7	
x_1	1	0	0	0	-1/160	0	0	-3/8	1	0	3
x_2	0	1	0	0	5591/33760	0	0	-107/1688	-53/633	0	316/211
x_3	0	0	1	0	-21939/33760	0	0	1703/1688	-103/633	0	849/211
u_1	0	0	0	1	13/2110	0	0	78/211	-528/211	0	208/211
u_3	0	0	0	0	1065/1688	1	0	-905/422	100/633	0	200/211
u_4	0	0	0	0	3/3376	0	1	45/844	-206/211	0	30/211
u_7	0	0	0	0	-213/13504	0	0	-1703/1688	-5/1266	1	-5/211
	0	0	0	0	19969/4220	0	0	827/211	1384/633	0	43280/211

	x_1	x_2	x_3	u_1	u_2	u_3	u_4	u_5	u_6	u_7	
x_1	1	0	0	0	-1/160	0	0	-3/8	1	0	3
x_2	0	1	0	0	5591/33760	0	0	-107/1688	-53/633	0	316/211
x_3	0	0	1	0	-21939/33760	0	0	1703/1688	-103/633	0	849/211
u_1	0	0	0	1	13/2110	0	0	78/211	-528/211	0	208/211
u_3	0	0	0	0	1065/1688	1	0	-905/422	100/633	0	200/211
u_4	0	0	0	0	3/3376	0	1	45/844	-206/211	0	30/211
$\leftarrow u_7$	0	0	0	0	-213/13504	0	0	-1703/1688	-5/1266	1	-5/211
	0	0	0	0	19969/4220	0	0	827/211	1384/633	0	43280/211

	x_1	x_2	x_3	u_1	u_2	u_3	u_4	u_5	u_6	u_7	
x_1	1	0	0	0	-1/160	0	0	-3/8	1	0	3
x_2	0	1	0	0	5591/33760	0	0	-107/1688	-53/633	0	316/211
x_3	0	0	1	0	-21939/33760	0	0	1703/1688	-103/633	0	849/211
u_1	0	0	0	1	13/2110	0	0	78/211	-528/211	0	208/211
u_3	0	0	0	0	1065/1688	1	0	-905/422	100/633	0	200/211
u_4	0	0	0	0	3/3376	0	1	45/844	-206/211	0	30/211
u_7	0	0	0	0	-213/13504	0	0	-1703/1688	-5/1266	1	-5/211
	0	0	0	0	19969/4220	0	0	827/211	1384/633	0	43280/211



x_1 x_2 x_3 u_1 u_2 u_3 u_4 u_5 u_6 u_7

x_1	1	0	0	0	-1/160	0	0	-3/8	1	0	3
x_2	0	1	0	0	5591/33760	0	0	-107/1688	-53/633	0	316/211
x_3	0	0	1	0	-21939/33760	0	0	1703/1688	-103/633	0	849/211
u_1	0	0	0	1	13/2110	0	0	78/211	-528/211	0	208/211
u_3	0	0	0	0	1065/1688	1	0	-905/422	100/633	0	200/211
u_4	0	0	0	0	3/3376	0	1	45/844	-206/211	0	30/211
	0	0	0	0	213/13624	0	0	1	20/5109	-1688/1703	40/1703
	0	0	0	0	19969/4220	0	0	827/211	1384/633	0	43280/211



	x_1	x_2	x_3	u_1	u_2	u_3	u_4	u_5	u_6	u_7	
x_1	1	0	0	0	-211/544960	0	0	0	3411/3406	-633/1703	5124/1703
x_2	0	1	0	0	90791/544960	0	0	0	-853/10218	-107/1703	2553/1703
x_3	0	0	1	0	-213/320	0	0	0	-1/6	1	4
u_1	0	0	0	1	1/2620	0	0	0	-328/131	48/131	128/131
u_3	0	0	0	0	18105/27248	1	0	0	850/5109	-3620/1703	1700/1703
u_4	0	0	0	0	3/54496	0	1	0	-1663/1703	90/1703	240/1703
u_5	0	0	0	0	213/13624	0	0	1	20/5109	-1688/1703	40/1703
	0	0	0	0	318169/68120	0	0	0	11092/5109	6616/1703	349160/1703

	x_1	x_2	x_3	u_1	u_2	u_3	u_4	u_5	u_6	u_7	
x_1	1	0	0	0	-211/544960	0	0	0	3411/3406	-633/1703	5124/1703
x_2	0	1	0	0	90791/544960	0	0	0	-853/10218	-107/1703	2553/1703
x_3	0	0	1	0	-213/320	0	0	0	-1/6	1	4
u_1	0	0	0	1	1/2620	0	0	0	-328/131	48/131	128/131
u_3	0	0	0	0	18105/27248	1	0	0	850/5109	-3620/1703	1700/1703
u_4	0	0	0	0	3/54496	0	1	0	-1663/1703	90/1703	240/1703
u_5	0	0	0	0	213/13624	0	0	1	20/5109	-1688/1703	40/1703
	0	0	0	0	318169/68120	0	0	0	11092/5109	6616/1703	349160/1703

Current BFS: $(5124/1703, 2553/1703, 4) = (3.008808, 1.499119, 4.000000)$