Primal Problem:

Maximize $z = x_1 + x_2 + x_3$
subject to

$$\begin{aligned} 2x_1 - x_2 + x_3 &= 4 \\ x_1 + x_2 \phantom{{}+x_3} &\geq 2 \\ 3x_1 + 2x_2 \phantom{{}+x_3} &\leq 7 \end{aligned}$$

$x_1, x_2, x_3 \geq 0$

Note: We negate second equation when forming dual problem.

Dual Problem:

Minimize $z' = 4w_1 - 2w_2 + 7w_3$
subject to

$$\begin{aligned} 2w_1 - w_2 + 3w_3 &\geq 1 \\ -w_1 - w_2 + 2w_3 &\geq 1 \\ w_1 &\geq 1 \end{aligned}$$

w_1 unrestricted; $w_2, w_3 \geq 0$

Canonical form of Primal Problem:

Maximize $z = x_1 + x_2 + x_3$
subject to

$$\begin{aligned} 2x_1 - \ x_2 + x_3 \qquad\qquad &= 4 \\ x_1 + \ x_2 \qquad - x_4 \qquad &= 2 \\ 3x_1 + 2x_2 \qquad\qquad + x_5 &= 7 \end{aligned}$$

$x_1, x_2, x_3, x_4, x_5 \geq 0$

Slack variables: x_4, x_5

Phase 1 Problem:

Minimize $z'' = y_1$
subject to

$$
\begin{aligned}
2x_1 - \ x_2 + x_3 \qquad\qquad\qquad\qquad &= 4 \\
x_1 + \ x_2 \qquad - x_4 \qquad + y_1 &= 2 \\
3x_1 + 2x_2 \qquad\qquad + x_5 \qquad &= 7
\end{aligned}
$$

$x_1, x_2, x_3, x_4, x_5, y_1 \geq 0$

Slack variables: x_4, x_5 ; Artificial variables: y_1

No need for artificial variable in row 1, since x_3 can be initial basic variable!

	x1	x2	x3	x4	x5	y1	
x3	2	-1	1	0	0	0	4
y1	1	1	0	-1	0	1	2
x5	3	2	0	0	1	0	7
	0	0	0	0	0	1	0

	x1	x2	x3	x4	x5	y1	
x3	2	-1	1	0	0	0	4
y1	1	1	0	-1	0	1	2
x5	3	2	0	0	1	0	7
	0	0	0	0	0	1	0

	x1	x2	x3	x4	x5	y1	
x3	2	-1	1	0	0	0	4
y1	1	1	0	-1	0	1	2
x5	3	2	0	0	1	0	7
	-1	-1	0	1	0	0	-2

	x1	x2	x3	x4	x5	y1	
x3	2	-1	1	0	0	0	4
y1	1	1	0	-1	0	1	2
x5	3	2	0	0	1	0	7
	-1	-1	0	1	0	0	-2

	x1	x2	x3	x4	x5	y1	
x3	2	-1	1	0	0	0	4
y1	1	1	0	-1	0	1	2
x5	3	2	0	0	1	0	7
	-1	-1	0	1	0	0	-2

	x1	x2	x3	x4	x5	y1	
x3	0	-3	1	2	0	-2	0
x1	1	1	0	-1	0	1	2
x5	0	-1	0	3	1	-3	1
	0	0	0	0	0	1	0

	x1	x2	x3	x4	x5	y1	
x3	0	-3	1	2	0	-2	0
x1	1	1	0	-1	0	1	2
x5	0	-1	0	3	1	-3	1
	-1	-1	-1	0	0	0	0

	x1	x2	x3	x4	x5	y1	
x3	0	-3	1	2	0	-2	0
x1	1	1	0	-1	0	1	2
x5	0	-1	0	3	1	-3	1
	-1	-1	-1	0	0	0	0

	x1	x2	x3	x4	x5	y1	
x3	0	-3	1	2	0	-2	0
x1	1	1	0	-1	0	1	2
x5	0	-1	0	3	1	-3	1
	0	-3	0	1	0	-1	2

	x1	x2	x3	x4	x5	y1	
x3	0	-3	1	2	0	-2	0
x1	1	1	0	-1	0	1	2
x5	0	-1	0	3	1	-3	1
	0	-3	0	1	0	-1	2

	x1	x2 ↓	x3	x4	x5	y1	
x3	0	-3	1	2	0	-2	0
← x1	1	1	0	-1	0	1	2
x5	0	-1	0	3	1	-3	1
	0	-3	0	1	0	-1	2

	x1	x2	x3	x4	x5	y1	
x3	3	0	1	-1	0	1	6
x2	1	1	0	-1	0	1	2
x5	1	0	0	2	1	-2	3
	3	0	0	-2	0	2	8

	x1	x2	x3	x4	x5	y1	
x3	3	0	1	-1	0	1	6
x2	1	1	0	-1	0	1	2
x5	1	0	0	2	1	-2	3
	3	0	0	-2	0	2	8

	x1	x2	x3	x4	x5	y1	
x3	3	0	1	-1	0	1	6
x2	1	1	0	-1	0	1	2
x5	1	0	0	2	1	-2	3
	3	0	0	-2	0	2	8

	x1	x2	x3	x4	x5	y1	
x3	3	0	1	-1	0	1	6
x2	1	1	0	-1	0	1	2
←	1/2	0	0	1	1/2	-1	3/2
	3	0	0	-2	0	2	8

	x1	x2	x3	x4	x5	y1	
x3	7/2	0	1	0	1/2	0	15/2
x2	3/2	1	0	0	1/2	0	7/2
x4	1/2	0	0	1	1/2	-1	3/2
	4	0	0	0	1	0	11

	x_1	x_2	x_3	x_4	x_5	y_1	
x_3	7/2	0	1	0	1/2	0	15/2
x_2	3/2	1	0	0	1/2	0	7/2
x_4	1/2	0	0	1	1/2	-1	3/2
	4	0	0	0	1	0	11

Opt. sol. to Primal Problem: $\widetilde{x} = (0, \frac{7}{2}, \frac{15}{2})$; $z(\widetilde{x}) = 11$

c^T+ final obj. row: $(1,1,1,0,0,0) + (4,0,0,0,1,0) = (5,1,1,0,1,0)$

Opt. sol. to Dual of Canonical Problem: $\widehat{w} = (1,0,1)$ (coefs. of x_3, y_1, x_5)

Opt. sol. to Dual of Primal Problem: $\widetilde{w} = (1,-0,1) = (1,0,1)$

Check: $\widetilde{x}$ and $\widetilde{w}$ are feasible solutions and $z'(\widetilde{w}) = 11 = z(\widetilde{x})$.

Add new constraint

Add constraint $x_3 \leq 7$ to Primal Problem.

Tableau represeinting $\widetilde{x} = (0, \frac{7}{2}, \frac{15}{2})$:

	x_1	x_2	x_3	x_4	x_5	
x_3	7/2	0	1	0	1/2	15/2
x_2	3/2	1	0	0	1/2	7/2
x_4	1/2	0	0	1	1/2	3/2
	4	0	0	0	1	11

Add slack variable x_6 and constraint $x_3 + x_6 = 7$

	x1	x2	x3	x4	x5	x6	
	7/2	0	1	0	1/2	0	15/2
x2	3/2	1	0	0	1/2	0	7/2
x4	1/2	0	0	1	1/2	0	3/2
x6	0	0	1	0	0	1	7
	4	0	0	0	1	0	11

	x1	x2	x3	x4	x5	x6	
	7/2	0	1	0	1/2	0	15/2
x2	3/2	1	0	0	1/2	0	7/2
x4	1/2	0	0	1	1/2	0	3/2
x6	0	0	1	0	0	1	7
	4	0	0	0	1	0	11

	x1	x2	x3	x4	x5	x6	
x3	7/2	0	1	0	1/2	0	15/2
x2	3/2	1	0	0	1/2	0	7/2
x4	1/2	0	0	1	1/2	0	3/2
x6	-7/2	0	0	0	-1/2	1	-1/2
	4	0	0	0	1	0	11

	x1	x2	x3	x4	x5	x6	
x3	7/2	0	1	0	1/2	0	15/2
x2	3/2	1	0	0	1/2	0	7/2
x4	1/2	0	0	1	1/2	0	3/2
← x6	-7/2	0	0	0	-1/2	1	-1/2
	4	0	0	0	1	0	11

	x1	x2	x3	x4	x5	x6	
x3	7/2	0	1	0	1/2	0	15/2
x2	3/2	1	0	0	1/2	0	7/2
x4	1/2	0	0	1	1/2	0	3/2
← x6	-7/2	0	0	0	-1/2	1	-1/2
	4	0	0	0	1	0	11

	x1	x2	x3	x4	x5	x6	
x3	7/2	0	1	0	1/2	0	15/2
x2	3/2	1	0	0	1/2	0	7/2
x4	1/2	0	0	1	1/2	0	3/2
←	1	0	0	0	1/7	-2/7	1/7
	4	0	0	0	1	0	11

	x1	x2	x3	x4	x5	x6	
x3	0	0	1	0	0	1	7
x2	0	1	0	0	2/7	3/7	23/7
x4	0	0	0	1	3/7	1/7	10/7
x1	1	0	0	0	1/7	-2/7	1/7
	0	0	0	0	3/7	8/7	73/7

Final tableau for Revised Primal Problem:

	x_1	x_2	x_3	x_4	x_5	x_6	
x_3	0	0	1	0	0	1	7
x_2	0	1	0	0	2/7	3/7	23/7
x_4	0	0	0	1	3/7	1/7	10/7
x_1	1	0	0	0	1/7	-2/7	1/7
	0	0	0	0	3/7	8/7	73/7

Optimal solution for Revised Primal Problem: $(x_1, x_2, x_3) = (\frac{1}{7}, \frac{23}{7}, 7)$