

Primal Problem:

Maximize $z = 3x_2 - x_3 - x_4 - x_5 - x_6$

subject to

$$x_1 + x_3 + x_5 = 1$$

$$x_2 + x_3 + x_4 = 1$$

$$x_1 + x_2 + x_6 \leq 2$$

$$x_3 + x_4 + x_5 + 2x_6 \geq 1$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

Dual Problem:

Minimize $z' = w_1 + w_2 + 2w_3 - w_4$
subject to

$$w_1 + w_3 \geq 0$$

$$w_2 + w_3 \geq 3$$

$$w_1 + w_2 - w_4 \geq -1$$

$$w_2 - w_4 \geq -1$$

$$w_1 - w_4 \geq -1$$

$$w_3 - 2w_4 \geq -1$$

$$w_1, w_2 \text{ unrestricted; } w_3, w_4 \geq 0$$

Phase 1 Problem:

Minimize $z'' = y_1 + y_2 + y_3$
subject to

$$x_1 + x_3 + x_5 + y_1 = 1$$

$$x_2 + x_3 + x_4 + y_2 = 1$$

$$x_1 + x_2 + x_6 + x_7 = 2$$

$$x_3 + x_4 + x_5 + 2x_6 - x_8 + y_3 = 1$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, y_1, y_2, y_3 \geq 0$$

Slack variables: x_7, x_8 ; Artificial variables: y_1, y_2, y_3

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
y1	1	0	1	0	1	0	0	0	1	0	0	1
y2	0	1	1	1	0	0	0	0	0	1	0	1
x7	1	1	0	0	0	1	1	0	0	0	0	2
y3	0	0	1	1	1	2	0	-1	0	0	1	1
	0	0	0	0	0	0	0	0	1	1	1	0

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
y1	1	0	1	0	1	0	0	0	1	0	0	1
y2	0	1	1	1	0	0	0	0	0	1	0	1
x7	1	1	0	0	0	1	1	0	0	0	0	2
y3	0	0	1	1	1	2	0	-1	0	0	1	1
	0	0	0	0	0	0	0	0	1	1	1	0

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
y1	1	0	1	0	1	0	0	0	1	0	0	1
y2	0	1	1	1	0	0	0	0	0	1	0	1
x7	1	1	0	0	0	1	1	0	0	0	0	2
y3	0	0	1	1	1	2	0	-1	0	0	1	1
	-1	-1	-3	-2	-2	-2	0	1	0	0	0	-3



	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
y1	1	0	1	0	1	0	0	0	1	0	0	1
y2	0	1	1	1	0	0	0	0	0	1	0	1
x7	1	1	0	0	0	1	1	0	0	0	0	2
y3	0	0	1	1	1	2	0	-1	0	0	1	1
	-1	-1	-3	-2	-2	-2	0	1	0	0	0	-3

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
y1	1	0	1	0	1	0	0	0	1	0	0	1
y2	0	1	1	1	0	0	0	0	0	1	0	1
x7	1	1	0	0	0	1	1	0	0	0	0	2
← y3	0	0	1	1	1	2	0	-1	0	0	1	1
	-1	-1	-3	-2	-2	-2	0	1	0	0	0	-3

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
y1	1	0	0	-1	0	-2	0	1	1	0	-1	0
y2	0	1	0	0	-1	-2	0	1	0	1	-1	0
x7	1	1	0	0	0	1	1	0	0	0	0	2
x3	0	0	1	1	1	2	0	-1	0	0	1	1
	-1	-1	0	1	1	4	0	-2	0	0	3	0

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
y1	1	0	0	-1	0	-2	0	1	1	0	-1	0
y2	0	1	0	0	-1	-2	0	1	0	1	-1	0
x7	1	1	0	0	0	1	1	0	0	0	0	2
x3	0	0	1	1	1	2	0	-1	0	0	1	1
	-1	-1	0	1	1	4	0	-2	0	0	3	0

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
\leftarrow y1	1	0	0	-1	0	-2	0	1	1	0	-1	0
y2	0	1	0	0	-1	-2	0	1	0	1	-1	0
x7	1	1	0	0	0	1	1	0	0	0	0	2
x3	0	0	1	1	1	2	0	-1	0	0	1	1
	-1	-1	0	1	1	4	0	-2	0	0	3	0

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
x8	1	0	0	-1	0	-2	0	1	1	0	-1	0
y2	-1	1	0	1	-1	0	0	0	-1	1	0	0
x7	1	1	0	0	0	1	1	0	0	0	0	2
x3	1	0	1	0	1	0	0	0	1	0	0	1
	1	-1	0	-1	1	0	0	0	2	0	1	0



x1 x2 x3 x4 x5 x6 x7 x8 y1 y2 y3

x8

1 0 0 -1 0 -2 0 1 1 0 -1 0

y2

-1 1 0 1 -1 0 0 0 -1 1 0 0

x7

1 1 0 0 0 1 1 0 0 0 0 2

x3

1 0 1 0 1 0 0 0 1 0 0 1

1 -1 0 -1 1 0 0 0 2 0 1 0

↓

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
--	----	----	----	----	----	----	----	----	----	----	----	--

x8	1	0	0	-1	0	-2	0	1	1	0	-1	0
← y2	-1	1	0	1	-1	0	0	0	-1	1	0	0
x7	1	1	0	0	0	1	1	0	0	0	0	2
x3	1	0	1	0	1	0	0	0	1	0	0	1
	1	-1	0	-1	1	0	0	0	2	0	1	0

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
x8	1	0	0	-1	0	-2	0	1	1	0	-1	0
x2	-1	1	0	1	-1	0	0	0	-1	1	0	0
x7	2	0	0	-1	1	1	1	0	1	-1	0	2
x3	1	0	1	0	1	0	0	0	1	0	0	1
	0	0	0	0	0	0	0	0	1	1	1	0

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
x8	1	0	0	-1	0	-2	0	1	1	0	-1	0
x2	-1	1	0	1	-1	0	0	0	-1	1	0	0
x7	2	0	0	-1	1	1	1	0	1	-1	0	2
x3	1	0	1	0	1	0	0	0	1	0	0	1
	0	-3	1	1	1	1	0	0	0	0	0	0

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
x8	1	0	0	-1	0	-2	0	1	1	0	-1	0
x2	-1	1	0	1	-1	0	0	0	-1	1	0	0
x7	2	0	0	-1	1	1	1	0	1	-1	0	2
x3	1	0	1	0	1	0	0	0	1	0	0	1
	0	-3	1	1	1	1	0	0	0	0	0	0

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
x8	1	0	0	-1	0	-2	0	1	1	0	-1	0
x2	-1	1	0	1	-1	0	0	0	-1	1	0	0
x7	2	0	0	-1	1	1	1	0	1	-1	0	2
x3	1	0	1	0	1	0	0	0	1	0	0	1
	-4	0	0	4	-3	1	0	0	-4	3	0	-1



x1

x2

x3

x4

x5

x6

x7

x8

y1

y2

y3

x8

1

0

0

-1

0

-2

0

1

1

0

-1

0

x2

-1

1

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-1

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-1

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x7

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x3

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-4

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0

4

-3

1

0

0

-4

3

0

-1

↓
x1

x2

x3

x4

x5

x6

x7

x8

y1

y2

y3

← x8

1

0

0

-1

0

-2

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1

1

0

-1

0

x2

-1

1

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-1

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-1

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x7

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-1

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-1

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x3

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1

0

0

0

1

0

0

1

-4

0

0

4

-3

1

0

0

-4

3

0

-1

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
x1	1	0	0	-1	0	-2	0	1	1	0	-1	0
x2	0	1	0	0	-1	-2	0	1	0	1	-1	0
x7	0	0	0	1	1	5	1	-2	-1	-1	2	2
x3	0	0	1	1	1	2	0	-1	0	0	1	1
	0	0	0	0	-3	-7	0	4	0	3	-4	-1

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
x1	1	0	0	-1	0	-2	0	1	1	0	-1	0
x2	0	1	0	0	-1	-2	0	1	0	1	-1	0
x7	0	0	0	1	1	5	1	-2	-1	-1	2	2
x3	0	0	1	1	1	2	0	-1	0	0	1	1
	0	0	0	0	-3	-7	0	4	0	3	-4	-1

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
x1	1	0	0	-1	0	-2	0	1	1	0	-1	0
x2	0	1	0	0	-1	-2	0	1	0	1	-1	0
x7	0	0	0	1	1	5	1	-2	-1	-1	2	2
x3	0	0	1	1	1	2	0	-1	0	0	1	1
	0	0	0	0	-3	-7	0	4	0	3	-4	-1

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
x1	1	0	0	-1	0	-2	0	1	1	0	-1	0
x2	0	1	0	0	-1	-2	0	1	0	1	-1	0
←	0	0	0	1/5	1/5	1	1/5	-2/5	-1/5	-1/5	2/5	2/5
x3	0	0	1	1	1	2	0	-1	0	0	1	1
	0	0	0	0	-3	-7	0	4	0	3	-4	-1

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
x1	1	0	0	$-3/5$	$2/5$	0	$2/5$	$1/5$	$3/5$	$-2/5$	$-1/5$	$4/5$
x2	0	1	0	$2/5$	$-3/5$	0	$2/5$	$1/5$	$-2/5$	$3/5$	$-1/5$	$4/5$
x6	0	0	0	$1/5$	$1/5$	1	$1/5$	$-2/5$	$-1/5$	$-1/5$	$2/5$	$2/5$
x3	0	0	1	$3/5$	$3/5$	0	$-2/5$	$-1/5$	$2/5$	$2/5$	$1/5$	$1/5$
	0	0	0	$7/5$	$-8/5$	0	$7/5$	$6/5$	$-7/5$	$8/5$	$-6/5$	$9/5$

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
x1	1	0	0	$-3/5$	$2/5$	0	$2/5$	$1/5$	$3/5$	$-2/5$	$-1/5$	$4/5$
x2	0	1	0	$2/5$	$-3/5$	0	$2/5$	$1/5$	$-2/5$	$3/5$	$-1/5$	$4/5$
x6	0	0	0	$1/5$	$1/5$	1	$1/5$	$-2/5$	$-1/5$	$-1/5$	$2/5$	$2/5$
x3	0	0	1	$3/5$	$3/5$	0	$-2/5$	$-1/5$	$2/5$	$2/5$	$1/5$	$1/5$
	0	0	0	$7/5$	$-8/5$	0	$7/5$	$6/5$	$-7/5$	$8/5$	$-6/5$	$9/5$

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
x1	1	0	0	$-3/5$	$2/5$	0	$2/5$	$1/5$	$3/5$	$-2/5$	$-1/5$	$4/5$
x2	0	1	0	$2/5$	$-3/5$	0	$2/5$	$1/5$	$-2/5$	$3/5$	$-1/5$	$4/5$
x6	0	0	0	$1/5$	$1/5$	1	$1/5$	$-2/5$	$-1/5$	$-1/5$	$2/5$	$2/5$
← x3	0	0	1	$3/5$	$3/5$	0	$-2/5$	$-1/5$	$2/5$	$2/5$	$1/5$	$1/5$
	0	0	0	$7/5$	$-8/5$	0	$7/5$	$6/5$	$-7/5$	$8/5$	$-6/5$	$9/5$

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
x1	1	0	0	$-3/5$	$2/5$	0	$2/5$	$1/5$	$3/5$	$-2/5$	$-1/5$	$4/5$
x2	0	1	0	$2/5$	$-3/5$	0	$2/5$	$1/5$	$-2/5$	$3/5$	$-1/5$	$4/5$
x6	0	0	0	$1/5$	$1/5$	1	$1/5$	$-2/5$	$-1/5$	$-1/5$	$2/5$	$2/5$
	0	0	$5/3$	1	1	0	$-2/3$	$-1/3$	$2/3$	$2/3$	$1/3$	$1/3$
	0	0	0	$7/5$	$-8/5$	0	$7/5$	$6/5$	$-7/5$	$8/5$	$-6/5$	$9/5$

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
x1	1	0	$-2/3$	-1	0	0	$2/3$	$1/3$	$1/3$	$-2/3$	$-1/3$	$2/3$
x2	0	1	1	1	0	0	0	0	0	1	0	1
x6	0	0	$-1/3$	0	0	1	$1/3$	$-1/3$	$-1/3$	$-1/3$	$1/3$	$1/3$
x5	0	0	$5/3$	1	1	0	$-2/3$	$-1/3$	$2/3$	$2/3$	$1/3$	$1/3$
	0	0	$8/3$	3	0	0	$1/3$	$2/3$	$-1/3$	$8/3$	$-2/3$	$7/3$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	y_1	y_2	y_3	
x_1	1	0	-2/3	-1	0	0	2/3	1/3	1/3	-2/3	-1/3	2/3
x_2	0	1	1	1	0	0	0	0	0	1	0	1
x_6	0	0	-1/3	0	0	1	1/3	-1/3	-1/3	-1/3	1/3	1/3
x_5	0	0	5/3	1	1	0	-2/3	-1/3	2/3	2/3	1/3	1/3
	0	0	8/3	3	0	0	1/3	2/3	-1/3	8/3	-2/3	7/3

Opt. sol. to Primal Problem: $\tilde{x} = (\frac{2}{3}, 1, 0, 0, \frac{1}{3}, \frac{1}{3})$

$c^T +$ final obj. row = $(0, 3, \frac{5}{3}, 2, -1, -1, \frac{1}{3}, \frac{2}{3}, -\frac{1}{3}, \frac{8}{3}, -\frac{2}{3})$

Opt. sol. to Dual of Canonical: $\hat{w} = (-\frac{1}{3}, \frac{8}{3}, \frac{1}{3}, -\frac{2}{3})$ (coefs. of y_1, y_2, x_7, y_3)

Opt. sol. to Dual of Primal: $\tilde{w} = (-\frac{1}{3}, \frac{8}{3}, \frac{1}{3}, \frac{2}{3})$

Check: \tilde{x} and \tilde{w} are feasible solutions and $z(\tilde{x}) = \frac{7}{3} = z'(\tilde{w})$.

Rerun two-phase algorithm.

Stop Phase 1 as soon as $z'' = y_1 + y_2 + y_3$ reaches zero.

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
y1	1	0	1	0	1	0	0	0	1	0	0	1
y2	0	1	1	1	0	0	0	0	0	1	0	1
x7	1	1	0	0	0	1	1	0	0	0	0	2
y3	0	0	1	1	1	2	0	-1	0	0	1	1
	0	0	0	0	0	0	0	0	1	1	1	0

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
y1	1	0	1	0	1	0	0	0	1	0	0	1
y2	0	1	1	1	0	0	0	0	0	1	0	1
x7	1	1	0	0	0	1	1	0	0	0	0	2
y3	0	0	1	1	1	2	0	-1	0	0	1	1
	0	0	0	0	0	0	0	0	1	1	1	0

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
y1	1	0	1	0	1	0	0	0	1	0	0	1
y2	0	1	1	1	0	0	0	0	0	1	0	1
x7	1	1	0	0	0	1	1	0	0	0	0	2
y3	0	0	1	1	1	2	0	-1	0	0	1	1
	-1	-1	-3	-2	-2	-2	0	1	0	0	0	-3



x1 x2 x3 x4 x5 x6 x7 x8 y1 y2 y3

y1	1	0	1	0	1	0	0	0	1	0	0	1
y2	0	1	1	1	0	0	0	0	0	1	0	1
x7	1	1	0	0	0	1	1	0	0	0	0	2
y3	0	0	1	1	1	2	0	-1	0	0	1	1
	-1	-1	-3	-2	-2	-2	0	1	0	0	0	-3

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
y1	1	0	1	0	1	0	0	0	1	0	0	1
y2	0	1	1	1	0	0	0	0	0	1	0	1
x7	1	1	0	0	0	1	1	0	0	0	0	2
← y3	0	0	1	1	1	2	0	-1	0	0	1	1
	-1	-1	-3	-2	-2	-2	0	1	0	0	0	-3

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
y1	1	0	0	-1	0	-2	0	1	1	0	-1	0
y2	0	1	0	0	-1	-2	0	1	0	1	-1	0
x7	1	1	0	0	0	1	1	0	0	0	0	2
x3	0	0	1	1	1	2	0	-1	0	0	1	1
	-1	-1	0	1	1	4	0	-2	0	0	3	0

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
y1	1	0	0	-1	0	-2	0	1	1	0	-1	0
y2	0	1	0	0	-1	-2	0	1	0	1	-1	0
x7	1	1	0	0	0	1	1	0	0	0	0	2
x3	0	0	1	1	1	2	0	-1	0	0	1	1
	0	-3	1	1	1	1	0	0	0	0	0	0

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
y1	1	0	0	-1	0	-2	0	1	1	0	-1	0
y2	0	1	0	0	-1	-2	0	1	0	1	-1	0
x7	1	1	0	0	0	1	1	0	0	0	0	2
x3	0	0	1	1	1	2	0	-1	0	0	1	1
	0	-3	1	1	1	1	0	0	0	0	0	0

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
y1	1	0	0	-1	0	-2	0	1	1	0	-1	0
y2	0	1	0	0	-1	-2	0	1	0	1	-1	0
x7	1	1	0	0	0	1	1	0	0	0	0	2
x3	0	0	1	1	1	2	0	-1	0	0	1	1
	0	-3	0	0	0	-1	0	1	0	0	-1	-1



x1 x2 x3 x4 x5 x6 x7 x8 y1 y2 y3

y1	1	0	0	-1	0	-2	0	1	1	0	-1	0
y2	0	1	0	0	-1	-2	0	1	0	1	-1	0
x7	1	1	0	0	0	1	1	0	0	0	0	2
x3	0	0	1	1	1	2	0	-1	0	0	1	1
	0	-3	0	0	0	-1	0	1	0	0	-1	-1





x1 x2 x3 x4 x5 x6 x7 x8 y1 y2 y3

y1	1	0	0	-1	0	-2	0	1	1	0	-1	0
← y2	0	1	0	0	-1	-2	0	1	0	1	-1	0
x7	1	1	0	0	0	1	1	0	0	0	0	2
x3	0	0	1	1	1	2	0	-1	0	0	1	1
	0	-3	0	0	0	-1	0	1	0	0	-1	-1

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
y1	1	0	0	-1	0	-2	0	1	1	0	-1	0
x2	0	1	0	0	-1	-2	0	1	0	1	-1	0
x7	1	0	0	0	1	3	1	-1	0	-1	1	2
x3	0	0	1	1	1	2	0	-1	0	0	1	1
	0	0	0	0	-3	-7	0	4	0	3	-4	-1

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
y1	1	0	0	-1	0	-2	0	1	1	0	-1	0
x2	0	1	0	0	-1	-2	0	1	0	1	-1	0
x7	1	0	0	0	1	3	1	-1	0	-1	1	2
x3	0	0	1	1	1	2	0	-1	0	0	1	1
	0	0	0	0	-3	-7	0	4	0	3	-4	-1

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
 y1	1	0	0	-1	0	-2	0	1	1	0	-1	0
x2	0	1	0	0	-1	-2	0	1	0	1	-1	0
x7	1	0	0	0	1	3	1	-1	0	-1	1	2
x3	0	0	1	1	1	2	0	-1	0	0	1	1
	0	0	0	0	-3	-7	0	4	0	3	-4	-1

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
	-1/2	0	0	1/2	0	1	0	-1/2	-1/2	0	1/2	0
x2	0	1	0	0	-1	-2	0	1	0	1	-1	0
x7	1	0	0	0	1	3	1	-1	0	-1	1	2
x3	0	0	1	1	1	2	0	-1	0	0	1	1
	0	0	0	0	-3	-7	0	4	0	3	-4	-1

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
x6	-1/2	0	0	1/2	0	1	0	-1/2	-1/2	0	1/2	0
x2	-1	1	0	1	-1	0	0	0	-1	1	0	0
x7	5/2	0	0	-3/2	1	0	1	1/2	3/2	-1	-1/2	2
x3	1	0	1	0	1	0	0	0	1	0	0	1
	-7/2	0	0	7/2	-3	0	0	1/2	-7/2	3	-1/2	-1



x1

x2

x3

x4

x5

x6

x7

x8

y1

y2

y3

x6

-1/2

0

0

1/2

0

1

0

-1/2

-1/2

0

1/2

0

x2

-1

1

0

1

-1

0

0

0

-1

1

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x7

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-3/2

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x3

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-7/2

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-7/2

3

-1/2

-1



x1

x2

x3

x4

x5

x6

x7

x8

y1

y2

y3

x6

-1/2

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x1

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x4

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x7

x8

y1

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1

-7/2

0

0

7/2

-3

0

0

1/2

-7/2

3

-1/2

-1

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
x6	0	0	0	$1/5$	$1/5$	1	$1/5$	$-2/5$	$-1/5$	$-1/5$	$2/5$	$2/5$
x2	0	1	0	$2/5$	$-3/5$	0	$2/5$	$1/5$	$-2/5$	$3/5$	$-1/5$	$4/5$
x1	1	0	0	$-3/5$	$2/5$	0	$2/5$	$1/5$	$3/5$	$-2/5$	$-1/5$	$4/5$
x3	0	0	1	$3/5$	$3/5$	0	$-2/5$	$-1/5$	$2/5$	$2/5$	$1/5$	$1/5$
	0	0	0	$7/5$	$-8/5$	0	$7/5$	$6/5$	$-7/5$	$8/5$	$-6/5$	$9/5$

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
x6	0	0	0	1/5	1/5	1	1/5	-2/5	-1/5	-1/5	2/5	2/5
x2	0	1	0	2/5	-3/5	0	2/5	1/5	-2/5	3/5	-1/5	4/5
x1	1	0	0	-3/5	2/5	0	2/5	1/5	3/5	-2/5	-1/5	4/5
x3	0	0	1	3/5	3/5	0	-2/5	-1/5	2/5	2/5	1/5	1/5
	0	0	0	7/5	-8/5	0	7/5	6/5	-7/5	8/5	-6/5	9/5

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
x6	0	0	0	1/5	1/5	1	1/5	-2/5	-1/5	-1/5	2/5	2/5
x2	0	1	0	2/5	-3/5	0	2/5	1/5	-2/5	3/5	-1/5	4/5
x1	1	0	0	-3/5	2/5	0	2/5	1/5	3/5	-2/5	-1/5	4/5
← x3	0	0	1	3/5	3/5	0	-2/5	-1/5	2/5	2/5	1/5	1/5
	0	0	0	7/5	-8/5	0	7/5	6/5	-7/5	8/5	-6/5	9/5

x1 x2 x3 x4 x5 x6 x7 x8 y1 y2 y3



x6

0 0 0 1/5 1/5 1 1/5 -2/5 -1/5 -1/5 2/5 2/5

x2

0 1 0 2/5 -3/5 0 2/5 1/5 -2/5 3/5 -1/5 4/5

x1

1 0 0 -3/5 2/5 0 2/5 1/5 3/5 -2/5 -1/5 4/5



0 0 5/3 1 1 0 -2/3 -1/3 2/3 2/3 1/3 1/3

0 0 0 7/5 -8/5 0 7/5 6/5 -7/5 8/5 -6/5 9/5

	x1	x2	x3	x4	x5	x6	x7	x8	y1	y2	y3	
x6	0	0	-1/3	0	0	1	1/3	-1/3	-1/3	-1/3	1/3	1/3
x2	0	1	1	1	0	0	0	0	0	1	0	1
x1	1	0	-2/3	-1	0	0	2/3	1/3	1/3	-2/3	-1/3	2/3
x5	0	0	5/3	1	1	0	-2/3	-1/3	2/3	2/3	1/3	1/3
	0	0	8/3	3	0	0	1/3	2/3	-1/3	8/3	-2/3	7/3