

Section 3.3, Problem 11:

Maximize $z = 3x_1 + x_2 + 3x_3 + x_4$

subject to

$$x_1 + x_2 + 4x_3 + x_4 \leq 6$$

$$2x_1 + 6x_3 + 2x_4 \geq 8$$

$$20x_1 + 2x_2 + 47x_3 + 11x_4 \leq 48$$

$$x \geq 0 \text{ in } \mathbb{R}^4$$

Dual problem:

Minimize $z' = 6w_1 - 8w_2 + 48w_3$

subject to

$$w_1 - 2w_2 + 20w_3 \geq 3$$

$$w_1 + 2w_3 \geq 1$$

$$4w_1 - 6w_2 + 47w_3 \geq 3$$

$$w_1 - 2w_2 + 11w_3 \geq 1$$

$$w \geq 0 \text{ in } \mathbb{R}^3$$

Phase 1:

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	y_1	
x_5	1	1	4	1	1	0	0	0	6
y_1	2	0	6	2	0	-1	0	1	8
x_7	20	2	47	11	0	0	1	0	48
	0	0	0	0	0	0	0	1	0

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	y_1	
x_5	1	1	4	1	1	0	0	0	6
y_1	2	0	6	2	0	-1	0	1	8
x_7	20	2	47	11	0	0	1	0	48
	-2	0	-6	-2	0	1	0	0	-8

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	y_1	
x_5	0	1	1	0	1	1/2	0	-1/2	2
x_4	1	0	3	1	0	-1/2	0	1/2	4
x_7	9	2	14	0	0	11/2	1	-11/2	4
	0	0	0	0	0	0	0	1	0

Phase 2:

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	y_1	
x_5	0	1	1	0	1	1/2	0	-1/2	2
x_4	1	0	3	1	0	-1/2	0	1/2	4
x_7	9	2	14	0	0	11/2	1	-11/2	4
	-3	-1	-3	-1	0	0	0	0	0

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	y_1	
x_5	0	1	1	0	1	1/2	0	-1/2	2
x_4	1	0	3	1	0	-1/2	0	1/2	4
x_7	9	2	14	0	0	11/2	1	-11/2	4
	-2	-1	0	0	0	-1/2	0	1/2	4

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	y_1	
x_5	0	1	1	0	1	1/2	0	-1/2	2
x_4	0	-2/9	13/9	1	0	-10/9	-1/9	10/9	32/9
x_1	1	2/9	14/9	0	0	11/18	1/9	-11/18	4/9
	0	-5/9	28/9	0	0	13/18	2/9	-13/18	44/9

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	y_1	
x_2	0	1	1	0	1	1/2	0	-1/2	2
x_4	0	0	5/3	1	2/9	-1	-1/9	1	4
x_1	1	0	4/3	0	-2/9	1/2	1/9	-1/2	0
	0	0	11/3	0	5/9	1	2/9	-1	6

Optimal solutions:

$$\tilde{x} = (0, 2, 0, 4)$$

$$\hat{w} = (5/9, -1, 2/9)$$

We negated the second inequality when we formed the dual problem.

$$\tilde{w} = (5/9, 1, 2/9)$$

$$z(\tilde{x}) = 6 = z'(\tilde{w})$$