Linear problem P:

Maximize $z = -x_1 + 2x_3 + x_4$
subject to

$$
\begin{array}{rcl}
3x_1 + x_2 - 2x_3 + 3x_4 & = & 10 \\
x_1 \qquad + \ \ x_3 - 2x_4 & \geq & -2 \\
x_1 \qquad + \ \ x_3 + \ \ x_4 & \geq & 10 \\
x_1 \qquad + 2x_3 + 3x_4 & \leq & 25
\end{array}
$$

$(x_1, x_2, x_3, x_4) \geq 0$ in $\mathbb{R}^4$

Dual problem:

Minimize $z^{\vee} = 10w_1 + 2w_2 - 10w_3 + 25w_4$
subject to

$$
\begin{array}{rcr}
3w_1 - w_2 - w_3 + w_4 & \geq & -1 \\
w_1 & \geq & 0 \\
-2w_1 - w_2 - w_3 + 2w_4 & \geq & 2 \\
3w_1 + 2w_2 - w_3 + 3w_4 & \geq & 1
\end{array}
$$

$w_2, w_3, w_4 \geq 0$; w_1 unrest

Canonical problem:

Maximize $z = -x_1 + 2x_3 + x_4$
subject to

$$
\begin{array}{rcl}
3x_1 + x_2 - 2x_3 + 3x_4 & = & 10 \\
-x_1 \qquad - \ \ x_3 + 2x_4 + u_1 & = & 2 \\
x_1 \qquad + \ \ x_3 + \ \ x_4 \qquad - u_2 & = & 10 \\
x_1 \qquad + 2x_3 + 3x_4 \qquad\qquad + u_3 & = & 25
\end{array}
$$

$(x_1, x_2, x_3, x_4, u_1, u_2, u_3) \geq 0$ in $\mathbb{R}^7$

Phase 1 problem:

Maximize $z' = -y_1$
subject to

$$\begin{aligned}
3x_1 + x_2 - 2x_3 + 3x_4 & & & & = 10 \\
-x_1 \quad\; - \; x_3 + 2x_4 + u_1 & & & & = 2 \\
x_1 \quad\; + \; x_3 + \; x_4 \quad - u_2 & & & + y_1 & = 10 \\
x_1 \quad\; + 2x_3 + 3x_4 & & + u_3 & & = 25
\end{aligned}$$

$(x_1, x_2, x_3, x_4, u_1, u_2, u_3, y_1) \geq 0$ in $\mathbb{R}^8$

	x_1	x_2	x_3	x_4	u_1	u_2	u_3	y_1	
x_2	3	1	-2	3	0	0	0	0	10
u_1	-1	0	-1	2	1	0	0	0	2
y_1	1	0	1	1	0	-1	0	1	10
u_3	1	0	2	3	0	0	1	0	25
	0	0	0	0	0	0	0	1	0

	x_1	x_2	x_3	x_4	u_1	u_2	u_3	y_1	
x_2	3	1	-2	3	0	0	0	0	10
u_1	-1	0	-1	2	1	0	0	0	2
y_1	1	0	1	1	0	-1	0	1	10
u_3	1	0	2	3	0	0	1	0	25
	0	0	0	0	0	0	0	1	0

	x_1	x_2	x_3	x_4	u_1	u_2	u_3	y_1	
x_2	3	1	-2	3	0	0	0	0	10
u_1	-1	0	-1	2	1	0	0	0	2
y_1	1	0	1	1	0	-1	0	1	10
u_3	1	0	2	3	0	0	1	0	25
	-1	0	-1	-1	0	1	0	0	-10

	x_1	x_2	x_3 ↓	x_4	u_1	u_2	u_3	y_1	
x_2	3	1	-2	3	0	0	0	0	10
u_1	-1	0	-1	2	1	0	0	0	2
y_1	1	0	1	1	0	-1	0	1	10
u_3	1	0	2	3	0	0	1	0	25
	-1	0	-1	-1	0	1	0	0	-10

	x_1	x_2	x_3 ↓	x_4	u_1	u_2	u_3	y_1	
x_2	3	1	-2	3	0	0	0	0	10
u_1	-1	0	-1	2	1	0	0	0	2
← y_1	1	0	(1)	1	0	-1	0	1	10
u_3	1	0	2	3	0	0	1	0	25
	-1	0	-1	-1	0	1	0	0	-10

	x_1	x_2	x_3	x_4	u_1	u_2	u_3	y_1	
x_2	5	1	0	5	0	-2	0	2	30
u_1	0	0	0	3	1	-1	0	1	12
x_3	1	0	1	1	0	-1	0	1	10
u_3	-1	0	0	1	0	2	1	-2	5
	0	0	0	0	0	0	0	1	0

	x_1	x_2	x_3	x_4	u_1	u_2	u_3	y_1	
x_2	5	1	0	5	0	-2	0	2	30
u_1	0	0	0	3	1	-1	0	1	12
x_3	1	0	1	1	0	-1	0	1	10
u_3	-1	0	0	1	0	2	1	-2	5
	1	0	-2	-1	0	0	0	0	0

Use original objective function: $z = -x_1 + 2x_3 + x_4$

	x_1	x_2	x_3	x_4	u_1	u_2	u_3	y_1	
x_2	5	1	0	5	0	-2	0	2	30
u_1	0	0	0	3	1	-1	0	1	12
x_3	1	0	1	1	0	-1	0	1	10
u_3	-1	0	0	1	0	2	1	-2	5
	1	0	-2	-1	0	0	0	0	0

	x_1	x_2	x_3	x_4	u_1	u_2	u_3	y_1	
x_2	5	1	0	5	0	-2	0	2	30
u_1	0	0	0	3	1	-1	0	1	12
x_3	1	0	1	1	0	-1	0	1	10
u_3	-1	0	0	1	0	2	1	-2	5
	3	0	0	1	0	-2	0	2	20

	x_1	x_2	x_3	x_4	u_1	u_2 ↓	u_3	y_1	
x_2	5	1	0	5	0	-2	0	2	30
u_1	0	0	0	3	1	-1	0	1	12
x_3	1	0	1	1	0	-1	0	1	10
u_3	-1	0	0	1	0	2	1	-2	5
	3	0	0	1	0	-2	0	2	20

	x_1	x_2	x_3	x_4	u_1	u_2 ↓	u_3	y_1	
x_2	5	1	0	5	0	-2	0	2	30
u_1	0	0	0	3	1	-1	0	1	12
x_3	1	0	1	1	0	-1	0	1	10
← u_3	-1	0	0	1	0	(2)	1	-2	5
	3	0	0	1	0	-2	0	2	20

	x_1	x_2	x_3	x_4	u_1	u_2 ↓	u_3	y_1	
x_2	5	1	0	5	0	-2	0	2	30
u_1	0	0	0	3	1	-1	0	1	12
x_3	1	0	1	1	0	-1	0	1	10
←	-1/2	0	0	1/2	0	(1)	1/2	-1	5/2
	3	0	0	1	0	-2	0	2	20

	x_1	x_2	x_3	x_4	u_1	u_2	u_3	y_1	
x_2	4	1	0	6	0	0	1	0	35
u_1	-1/2	0	0	7/2	1	0	1/2	0	29/2
x_3	1/2	0	1	3/2	0	0	1/2	0	25/2
u_2	-1/2	0	0	1/2	0	1	1/2	-1	5/2
	2	0	0	2	0	0	1	0	25

	x_1	x_2	x_3	x_4	u_1	u_2	u_3	y_1	
x_2	4	1	0	6	0	0	1	0	35
u_1	-1/2	0	0	7/2	1	0	1/2	0	29/2
x_3	1/2	0	1	3/2	0	0	1/2	0	25/2
u_2	-1/2	0	0	1/2	0	1	1/2	-1	5/2
	2	0	0	2	0	0	1	0	25

Optimal solution to P: $(0, 35, \frac{25}{2}, 0)$

$$c^T = (-1, 0, 2, 1, 0, 0, 0, 0)$$

$$\text{final obj row} + c^T = (1, 0, 2, 3, 0, 0, 1, 0)$$

Initial basic variables:

x_2, u_1, y_1, u_3

$\widehat{w} = (0, 0, 0, 1)$ (dual of canonical)

$\widetilde{w} = (0, 0, -0, 1)$ (dual of P)