

Problem:

Maximize $z = 2x_1 + x_2 + x_3$

subject to

$$3x_1 + x_2 + 3x_3 = 9$$

$$3x_1 - x_2 = 3$$

Solve with two-phase algorithm.

x1 x2 x3 y1 y2

y1

3	1	3	1	0	9
---	---	---	---	---	---

y2

3	-1	0	0	1	3
---	----	---	---	---	---

0	0	0	1	1	0
---	---	---	---	---	---

	x1	x2	x3	y1	y2	
y1	3	1	3	1	0	9
y2	3	-1	0	0	1	3
	0	0	0	1	1	0

	x1	x2	x3	y1	y2	
y1	3	1	3	1	0	9
y2	3	-1	0	0	1	3
	-6	0	-3	0	0	-12



x1

x2

x3

y1

y2

y1

3

1

3

1

0

9

y2

3

-1

0

0

1

3

-6

0

-3

0

0

-12



x1

x2

x3

y1

y2

y1

3

1

3

1

0

9

← y2

3

-1

0

0

1

3

-6

0

-3

0

0

-12

	3	1	3	1	0	9
	3	-1	0	0	1	3
	-6	0	-3	0	0	-12



x1

x2

x3

y1

y2

y1

3	1	3	1	0	9
1	-1/3	0	0	1/3	1
-6	0	-3	0	0	-12



x1 x2 x3 y1 y2

y1

0	2	3	1	-1	6
---	---	---	---	----	---

x1

1	-1/3	0	0	1/3	1
---	------	---	---	-----	---

0	-2	-3	0	2	-6
---	----	----	---	---	----



x1

x2

x3

y1

y2

y1

0

2

3

1

-1

6

x1

1

-1/3

0

0

1/3

1

0

-2

-3

0

2

-6

x1 x2 x3 y1 y2



	0	2	3	1	-1	6
x1	1	-1/3	0	0	1/3	1
	0	-2	-3	0	2	-6



x1 x2 x3 y1 y2



← x3

0	$\frac{2}{3}$	1	$\frac{1}{3}$	$-\frac{1}{3}$	2	
x1	1	$-\frac{1}{3}$	0	0	$\frac{1}{3}$	1
	0	-2	-3	0	2	-6

x1 x2 x3 y1 y2

x3

0

$2/3$

1

$1/3$

$-1/3$

2

x1

1

$-1/3$

0

0

$1/3$

1

0

0

0

1

1

0

x1 x2 x3

x3

0

$2/3$

1

2

x1

1

$-1/3$

0

1

-2

-1

-1

0

x1 x2 x3

x3

0

$2/3$

1

2

x1

1

$-1/3$

0

1

-2

-1

-1

0

x1 x2 x3

x3

0

$2/3$

1

2

x1

1

$-1/3$

0

1

0

-1

0

4



x1 x2 x3

x3

0	$2/3$	1	2
---	-------	---	---

x1

1	$-1/3$	0	1
---	--------	---	---

0	-1	0	4
---	----	---	---

x1 x2 x3



$\frac{2}{3}$

← x3

x1

0	$\frac{2}{3}$	1	2
1	$-\frac{1}{3}$	0	1
0	-1	0	4

x1 x2 x3



	0	1	$3/2$	3
x1	1	$-1/3$	0	1
	0	-1	0	4



x1

x1 x2 x3

x2

0

1

$3/2$

3

x1

1

0

$1/2$

2

0

0

$3/2$

7