

SOLUTION TO HW 5

2.1 9.

Theorem: \forall sets $A, B, C : (A \subset B \text{ and } B \subset C \text{ and } C \subset A) \Rightarrow (A = B \text{ and } B = C)$

Proof. Let A, B, C be sets.

Assume that $A \subset B$ and $B \subset C$ and $C \subset A$.

Since $B \subset C$ and $C \subset A$, it follows that $B \subset A$.

Since $A \subset B$ and $B \subset A$, we must have $A = B$.

Since $C \subset A$ and $A \subset B$, it follows that $C \subset B$.

Since $B \subset C$ and $C \subset B$, we must have $B = C$. □