Nathanson heights in finite vector spaces

Let p be a prime, and let $\mathbb{Z}_p$ denote the field of integers modulo p. The *Nathanson height* of a point $v \in \mathbb{Z}_p^n$ is the sum of the least nonnegative integer representatives of its coordinates. The Nathanson height of a subspace $V \subseteq \mathbb{Z}_p^n$ is the least Nathanson height of any of its nonzero points. In this talk, I will investigate the range of the Nathanson height function using a variety of techniques from additive combinatorics. In particular, I will show that on subspaces of $\mathbb{Z}_p^n$ of codimension one, the Nathanson height function can only take values about $p, p/2, p/3, \ldots$. I prove this by showing a similar result for the coheight on subsets of $\mathbb{Z}_p$, where the *coheight* of $A \subseteq \mathbb{Z}_p$ is the minimum number of times A must be added to itself so that the sum contains 0. I will also present some open questions and conjectures related to the Nathanson height, and indicate a few possible directions for future research.

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