

**NEW YORK NUMBER THEORY SEMINAR - ZOOM  
SUMMER, 2020**

Date: Friday, July 10, 2020, at 11:00 a.m. EDT  
Speaker: Shalom Eliahou, Université du Littoral Côte d'Opale, France  
Title: Iterated sumsets and Hilbert functions  
Abstract: Let  $A, B \subset \mathbf{Z}$ . Denote  $A + B = \{a + b \mid a \in A, b \in B\}$ , the *sumset* of  $A, B$ . For  $A = B$ , denote  $2A = A + A$ . More generally, for  $h \geq 2$ , denote  $hA = A + (h - 1)A$ , the  $h$ -fold *iterated sumset* of  $A$ . If  $A$  is finite, how does the sequence  $|hA|$  behave as  $h$  grows? This is a typical problem in additive combinatorics. In this talk, we focus on the following specific question: If  $|hA|$  is known, what can one say about  $|(h - 1)A|$  and  $|(h + 1)A|$ ? It is known that  $|(h - 1)A| \geq |hA|^{(h-1)/h}$ , a consequence of Plünnecke's inequality derived from graph theory. Here we propose a new approach, by modeling the sequence  $|hA|$  with the Hilbert function of a suitable standard graded algebra  $R(A)$ . We then apply Macaulay's 1927 theorem on the growth of Hilbert functions. This allows us to recover and strengthen Plünnecke's estimate on  $|(h - 1)A|$ . This is joint work with Eshita Mazumdar.

Zoom: <https://us02web.zoom.us/j/84573603409>  
Meeting ID: 845 7360 3409