Homework 1 Solutions, Math 477, Fall 2018

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Chapter 1 Problems:

5. If the first digit is 4, there are 2 choices for the second digit (it must be 0 or 1) and then 9 choices for the first digit, which must be chosen from $\{1, \ldots, 9\}$. Thus there are 18 such allowed area codes.

8b. The word "propose" has 2 p's, 2 o's and 1 each of r, s and e. There a 7 letters in total, and so there are

$$\frac{7!}{(2!)^2(1!)^3} = 1260 \ .$$

21. We must count the ways of taking 7 steps, exactly 4 of which are to the right, and exactly 3 of which are up. Once the 4 steps to the right are determined, everything is determined, so the number is the number of ways we can select 4 places out of 7 for the steps to the right (or equivalently, 3 places out of seven for the steps up). The answer is therefore

$$\binom{7}{4} = \binom{7}{3} = \frac{7!}{4!3!} = 35 \ .$$

32. This is just like a blotting problem where there are 8 voters and 6 candidate. The number of final vote tallies is

$$\binom{8+6-1}{6-1} = \binom{13}{5} = 1287 \ ,$$

and this is therefore the number of exit patterns the operator could see.

For the second part, we effectively have two independent elections for 6 candidates, one with 5 voters and one with 3 voters, These have

$$\binom{5+6-1}{6-1} = 252 \qquad \text{and} \qquad \binom{3+6-1}{6-1} = 56$$

possible outcomes respectively. The total is their product, namely 14112, which is, of course, more than before.

Chapter 1 Theoretical Exercises:

20. Let $k := n - \sum_{j=1}^r m_j$. This is how many ball are left when the required minimum numbers have been placed in each urn. At this point, assuming $k > 0$, we have choices. We seek r non-negative integer $\{n+1, \ldots, n_r\}$ such that $\sum_{j=1}^r n_j = k$. There are

$$\binom{k+r-1}{r-1}$$

$(n_1, \dots, n_r)$ such vectors.

22. By Clairault's Theorem, provided the partial derivatives are continuous, it does not matter in which order they are taken; all that matters is that there are n_j partial derivatives with respect to x_j, $n_j \geq 0$, for each $j = 1, \dots, n$ and $\sum_{j=1}^{n} n_j = r$. Let's be reasonable and assume Clairault's Theorem applies (though nothing is said about the regularity of f). Then the answer is

$$\binom{k+r-1}{n-1}$$

23. By one of our basic counting results, we know that there are exactly $\binom{n+\ell-1}{n-1}$ vectors $(x_1, \dots, x_n)$ of non-negative integers such that $\sum_{i=1}^{n} x_i = \ell$. Therefore, the number of vectors $(x_1, \dots, x_n)$ of non-negative integers such that $\sum_{i=1}^{n} x_i \leq k$. is

$$\sum_{\ell=0}^{k} \binom{n+\ell-1}{n-1} .$$