Homework 5 Solutions, Math 477, Fall 2018

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From the Problems in Chapter 4:

25: The sample space S is given by $S = \{(HH), (HT), (TH), (TT)\}$. By independence and the given information,

$$P(HH) = 0.42\ , \quad P(HT) = 0.18\ , \quad P(TH) = 0.28\ , \quad P(TT) = 0.12\ .$$

If X is the number of heads, the event $\{X = 1\}$ is given by $\{(HT), (TH)\}$ and so

$$P(\{X = 1\}) = 0.18 + 0.28 = 0.46\ .$$

Also,

$$\mathrm{E}(X) = 2 \cdot P(HH) + 1(P(HT) + P(TH)) + 0P(TT) = 1.28\ .$$

38: Since $\mathrm{Var}(X) = \mathrm{E}(X^2) - (\mathrm{E}(X))^2$, $5 = \mathrm{E}(X^2) - 1$, and so $\mathrm{E}(X^2) = 6$.

Now, $(2 + X)^2 = 2 + 4X + X^2$, and then by the linearity of expectation,

$$\mathrm{E}((2 + X)^2) = 2 + 4 + 6 = 12\ .$$

Next, since the constant random variable 4 is independent of X – it is independent of everything –, and has zero variance – it is not really random, we have

$$\mathrm{Var}(4 + 3X) = \mathrm{Var}(4) + 9\mathrm{Var}(X) = 45\ .$$

From the theoretical exercises in Chapter 4:

14: The proportion of families that have at least one child is $\sum_{n=1}^{\infty} \alpha p^n = \alpha \frac{p}{1-p}$, which is less than 1 by the information given on α. That is a randomly selected family with have no children with probability

$$1 - \alpha \frac{p}{1-p}\ .$$

Let E_n be the event that a family has n children. Let F be the event that a family has exactly k boys (and any number of girls). Then

$$P(F) = \sum_{n=k}^{\infty} P(F \cap E_n) = \sum_{n=k}^{\infty} P(E_n)P(F|E_n) = \alpha \sum_{n=k}^{\infty} p^n 2^{-n} \binom{n}{k}\ .$$