Practice Test Two, Math 477

November 18, 2018

NAME: _____

Circle problems to be graded: 1 2 3 4 5

1.	
2.	
3.	
4.	
5.	
Total	

1. If you buy a ticket in each of 50 lotteries, and you have a 1 in 100 chance of winning a prize in each of them, what is the (approximate) probability that you will win:

(a) at least once

(b) exactly once

(c) at least twice

2. A factory produces two types of coin. One is fair, and the other is biased; it comes up heads 55% of the time. We have a coin from the factory, but do not know what type it is. To test this, we toss the coin $1,000$ times. We will decide the coin is fair if the resulting numbers of heads is less than 525, and we will decide the coin is biased is the resulting numbers of heads is at least 525. There are two types of errors we could make: (1) Deciding the coin is fair when actually it is biased. and (2) deciding the coin is biased when actually it is fair. Determine the (approximate) probability of both types of error.

3. Let X be a random variable whose distribution is exponential with parameter λ. Define $Y := [X]$, where $[x]$ denotes the *integer part* of x; i.e., the largest integer that is not greater than x. Define $Z = X - Y = X - [X]$; this is the *fractional part* of X. For n a non-negative integer and $x \in (0, 1)$, compute each of

$$P(Y = n) \ , \qquad P(Z \leq x) \quad \text{and} \quad P(Y = n \text{ and } Z \leq x) \ .$$

Are Y And Z independent? Also, compute the variance of Z.

4. Three trucks break down on a road of length L, and the locations of the breakdowns are independent and uniformly distributed. For $0 < d < L/2$. compute the probability that no two trucks are within a distance d of one another.

5. Let X and Y be the Cartesian coordinates of a point chosen at random form the right half of the centered unit circle. That is, the joint probability density function of X and Y is

$$f(x,y) = \begin{cases} \frac{2}{\pi} & x^2 + y^2 \leq 1 \ , x > 0 \\ 0 & \text{otherwise} \end{cases} .$$

Let $U = \sqrt{X^2 + Y^2}$ and $V = \arctan(Y/X)$. Also, compute the joint probability density function of U and V. Compute the probability density function of $U + V$.