

Exercises in Logic¹

TOO LARGE

1. What is the relationship between algebraic geometry and complex analysis?
2. What is the relationship between combinatorial group theory and low dimensional topology?
3. What is quantization, really?
4. Turn the web into an artificial intelligence.
5. Define “game” robustly. Prove that this is a less general notion than “algorithm”. Discuss utility functions.
(This sounds quite odd, and I wish I had phrased it more clearly. This could be motivated by game theoretic approaches to evolution as well as issues in economics, and does not appear to distinguish the two, for some reason. A possible reference is [Papadimitriou 2001](#))
6. The dichotomy in mathematics between discrete and continuous quantities seems to correspond to the two main algorithmic structures in the human brain: visual analysis and language skills. Which is the chicken and which is the egg? (Ref: DeHaene.)
See also <https://micromath.wordpress.com/> et passim.
7. Are human language mechanisms optimal? (Chomsky)
Cf. <http://www.let.rug.nl/zwart/docs/minprogrev.pdf>
8. Logic with hormones and artificial intelligence.
9. The neurological basis of consciousness. Cf. Kasten, In Search of Memory. (Might be better placed under “Extra Credit” below.)
10. Inconsistency of physics. Is it true that any conventional mathematical theory which incorporates a theory of the very small and the very large must be logically inconsistent?

¹March 2007. Last revised 2017.

TOO SMALL

1. Classify the metrically homogeneous graphs.
 2. What is the p -adic version of o -minimality?
 3. Describe the saturated model of the theory of the free group.
 4. Make a torsion-free simple group of finite Morley rank, or at least export the problem to the combinatorial group theorists.
 5. When does a homogeneous structure for a finite relational language have the finite model property? More broadly, is there anything of interest in graph theory besides randomness and algebra?
 6. Can one determine whether a natural class of finite structures has an infinite antichain?
 7. Organize the proof of the classification of the finite simple groups.
 8. Similarly: Formalize chess endgame theory on an arbitrary rectangular board.
 9. What are the people working in representation theory of Lie groups trying to do, and is it possible?
 10. What is the theory of complex exponentiation? (Zilber)
 11. Graph reconstruction conjecture
 12. Find a definition of chess endgames which makes them PSPACE complete. (Chess is EXPTIME complete.) Cf. *Complexity, appeal and challenges of combinatorial games*, Aviezri S. Fraenkel, Theoretical Computer Science 313, 2004.
- Also <http://www.wisdom.weizmann.ac.il/~fraenkel/Papers/ICGA.pdf>

JUST RIGHT

Warm-ups

1. The complexity of graph isomorphism. (Cf. Immerman; and now Babai, which really takes this off the list.)
2. Hilbert's 10th problem over \mathbb{Q} .
3. Classify the ultrahomogeneous geodesic metric spaces. (Examples: metrically homogeneous graphs, Urysohn space) up to some notion of equivalence which in the discrete case is isomorphism.
4. How can one determine what a document is talking about?

Assignment

5. (Hodges) Study the semantic ideas of medieval Arab linguists outside the Aristotelian tradition.
6. What are mathematicians trying to do, and is it possible? (decidability, Borel reductions)
7. What is the language of probabilistic graph theory?
8. Develop a "gamer theory" and discuss the appropriate use of randomness in game design (agon/alea).

Ref.

http://onlyagame.typepad.com/only_a_game/2005/11/the_rituals_of_.html

Short form: <http://alturl.com/mnkcw>.

Extra Credit

9. The foundations of economics.
10. Legal logic; the uses of ambiguity (cf.: friction as a control mechanism).

Notes, 2017

- It didn't occur to me to ask for a better definition of "natural" in mathematics than we already have; Voevodsky gets a gold star for asking this and proposing an answer.
- Zilber's current take on the relationship between logic and physics is that logic should have something to say about the passage from the large finite to the continuous; or from a logical point of view, the passage from the hyperfinite to the continuous.
- I did not mention explicitly much of the actual on-going work here. That would probably require another, longer document, and regular updates.
- The occasion for this note (loosely speaking) was a panel discussion on "Connections of Logic with other parts of mathematics" with Martin Davis and Mirna Džamonja, moderated by David Linetsky, March 17, 2007, at St. John's University's Manhattan campus.

I thought it was fairly well hidden on my website, but I see that it has been found at least occasionally. But I have no way of knowing whether it is more popular than the note on bubble wrap.

- Older versions: [2007](#) [2016](#)