# Some ternary homogeneous structures

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# Concrete classification problems for ternary languages

! Homogeneous hypertournaments

? Homogeneous families of linear orders

Some examples and some questions.

(Slightly fuller version in ”TernaryProblems”)

# $t$-hypertournaments

**Definition**

$t$-ary, antisymmetric (exactly $\mathrm{Alt}_t$-symmetric).

# The classification problem

**Problem**

*Finitely many (boundedly many?) homogeneous $t$-hypertournaments for each $t$?*

Known for $t = 2$ and possible only because there is a unique $t$-type up to symmetry (no Henson construction).

**Problem**

*For $t \geq 3$, are there any infinite ones which are not $(t+1)$-constrained?*

(One for $t = 2$.)

# Even hypertournaments

## Definition

$t$ odd:
Parity of a $t$-hypertournament on a $(t+1)$-set: well-defined (take a linear order and count increasing $t$-tuples which are hyperarcs).
Even: All parities on $(t+1)$-sets are even.

## Proposition

*An even homogeneous $t$-hypertournament (so, $t$ odd) restricts to a homogeneous $(t-1)$-hypertournament, and is determined by the latter.*

# The case $t = 3$: Catalog

There is a unique homogeneous $t$-hypertournament $H_{t+1}$ of order $t+1$, $\mathrm{Aut}(H_{t+1}) = \mathrm{Alt}_{t+1}$.
Three 4-types: $H_4$, $C_4$ (circular order), $O_4$ (odd).

## Proposition

*A homogeneous 3-hypertournament has one of the following forms.*

- *Finite, order* $1, 2, 4, 8$ *(trivial;* $H_4$*;* $A(1, \mathbb{F}_8)$*) (W. Kantor, 1972).*
- *Generic cyclic order, realizes only type* $C_4$*.*
- *Generic even 3-hypertournament, omits* $O_4$*.*
- *Realizes* $C_4$*,* $O_4$*, omits* $H_4$*. [Generic exists—any others?]*
- *Realizes all 4-types [Generic exists—any others?]*

# Homogeneous FLO

## Definition

FLO: $(A, R(x, y, z))$: $R$ irreflexive relation; $R(a, x, y) =<_a$ linear on $A \setminus \{a\}$ and not derived from a constant order on $A$.

This seems like a class worth working out the Ramsey theory for, for any examples one can come up with, whether or not one has chances for a real classification. In any case the first step is to work out the "natural" examples and even this is not done systematically.

## Unfortunate Proposition

There are $2^{\aleph_0}$ homogeneous *FLO*.

# Construction of $2^{\aleph_0}$ homogeneous FLOs

Control the cyclic part of the relation.

- Build an infinite antichain of irreducible cyclic structures contained in FLOs
- Amalgamate without introducing new irreducible cyclic structures.

Antichain: derived from cyclic orders on at least 5 points by replacing triples $x < y < z$ with $x, y, z$ consecutive by $x, z, y$.
Amalgamation: $A_0 \subseteq A_0 \cup \{a_1\}, A_0 \cup \{a_2\}$:
make $A_0 <_{a_1} a_2$ and $A_0 <_{a_2} a_1$.

# So . . .

**Problem**

*How many homogeneous FLOs are there with trivial cyclic part?*

**Problem**

*What exactly are the 4-constrained homogeneous FLO's?*